Design for optimum performance and durability in demanding variable speed motor applications. D.C. motors have earned a reputation for dependability in severe operating conditions.
29.1. Motor Principle

An Electric motor is a machine which converts electric energy into mechanical energy. Its action is based on the principle that when a current-carrying conductor is placed in a magnetic field, it experiences a mechanical force whose direction is given by Fleming's Left-hand Rule and whose magnitude is given by \( F = BIL \) Newton.

![Principle of Motor](image)

Constructionally, there is no basic difference between a d.c. generator and a d.c. motor. In fact, the same d.c. machine can be used interchangeably as a generator or as a motor. D.C. motors are also like generators, shunt-wound or series-wound or compound-wound.

In Fig. 29.1 a part of multipolar d.c. motor is shown. When its field magnets are excited and its armature conductors are supplied with current from the supply mains, they experience a force tending to rotate the armature. Armature conductors under N-pole are assumed to carry current downwards (crosses) and those under S-poles, to carry current upwards (dots). By applying Fleming's Left-hand Rule, the direction of the force on each conductor can be found. It is shown by small arrows placed above each conductor. It will be seen that each conductor can be found. It will be seen that each conductor experiences a force \( F \) which tends to rotate the armature in anticlockwise direction. These forces collectively produce a driving torque which sets the armature rotating.

It should be noted that the function of a commutator in the motor is the same as in a generator. By reversing current in each conductor as it passes from one pole to another, it helps to develop a continuous and unidirectional torque.

29.2. Comparison of Generator and Motor Action

As said above, the same d.c. machine can be used, at least theoretically, interchangeably as a generator or as a motor. When operating as a generator, it is driven by a mechanical machine and it develops voltage which in turn produces a current flow in an electric circuit. When operating as a motor, it is supplied by electric current and it develops torque which in turn produces mechanical rotation.

Let us first consider its operation as a generator and see how exactly and through which agency, mechanical power is converted into electric power.

In Fig. 29.2 part of a generator whose armature is being driven clockwise by its prime mover is shown.

Fig. 29.2 (a) represents the fields set up independently by the main poles and the armature conductors like \( A \) in the figure. The resultant field or magnetic lines of flux are shown in Fig. 29.2 (b).
It is seen that there is a crowding of lines of flux on the right-hand side of A. These magnetic lines of flux may be likened to the rubber bands under tension. Hence, the bent lines of flux up a mechanical force on A much in the same way as the bent elastic rubber band of a catapult produces a mechanical force on the stone piece. It will be seen that this force is in a direction opposite to that of armature rotation. Hence, it is known as backward force or magnetic drag on the conductors. It is against this drag action on all armature conductor that the prime mover has to work. The work done in overcoming this opposition is converted into electric energy. Therefore, it should be clearly understood that it is only through the instrumentality of this magnetic drag that energy conversion is possible in a d.c. generator.

Next, suppose that the above d.c. machine is uncoupled from its prime mover and that current is sent through the armature conductors under a N-pole in the downward direction as shown in Fig. 29.3 (a). The conductors will again experience a force in the anticlockwise direction (Fleming’s Left hand Rule). Hence, the machine will start rotating anticlockwise, thereby developing a torque which can produce mechanical rotation. The machine is then said to be motoring.

As said above, energy conversion is not possible unless there is some opposition whose overcoming provides the necessary means for such conversion. In the case of a generator, it was the magnetic drag which provided the necessary opposition. But what is the equivalent of that drag in the case of a motor? Well, it is the back e.m.f. It is explained in this manner:

As soon as the armature starts rotating, dynamically (or motionally) induced e.m.f. is produced in the armature conductors. The direction of this induced e.m.f. as found by Fleming’s Right-hand Rule, is outwards i.e., in direct opposition to the applied voltage (Fig. 29.3 (b)). This is why it is known as back e.m.f. $E_b$ or counter e.m.f. Its value is the same as for the motionally induced e.m.f. in the generator i.e. $E_b = (ΦZN) \times (P/A)$ volts. The applied voltage $V$ has to be force current through the armature conductors against this back e.m.f. $E_b$. The electric work done in overcoming this opposition is converted into mechanical energy developed in the armature. Therefore, it is obvious that but for the production of this opposing e.m.f. energy conversion would not have been possible.

Now, before leaving this topic, let it be pointed out that in an actual motor with slotted armature, the torque is not due to mechanical force on the conductors themselves, but due to tangential pull on the armature teeth as shown in Fig. 29.4.

It is seen from Fig. 29.4 (a) that the main flux is concentrated in the form of tufts at the armature teeth while the armature flux is shown by the dotted lines embracing the armature slots. The effect of

In fact, it seems to be one of the fundamental laws of Nature that no energy conversion from one form to another is possible until there is some one to oppose the conversion. But for the presence of this opposition, there would simply be no energy conversion. In generators, opposition is provided by magnetic drag whereas in motors, back e.m.f. does this job. Moreover, it is only that part of the input energy which is used for overcoming this opposition that is converted into the other form.
armature flux on the main flux, as shown in Fig. 29.4 (b), is two-fold:

(i) It increases the flux on the left-hand side of the teeth and decreases it on the right-hand side, thus making the distribution of flux density across the tooth section unequal.

(ii) It inclines the direction of lines of force in the air-gap so that they are not radial but are disposed in a manner shown in Fig. 29.4 (b). The pull exerted by the poles on the teeth can now be resolved into two components. One is the tangential component $F_1$ and the other vertical component $F_2$. The vertical component $F_2$, when considered for all the teeth round the armature, adds up to zero. But the component $F_1$ is not cancelled and it is this tangential component which, acting on all the teeth, gives rise to the armature torque.

### 29.3. Significance of the Back e.m.f.

As explained in Art 29.2, when the motor armature rotates, the conductors also rotate and hence cut the flux. In accordance with the laws of electromagnetic induction, e.m.f. is induced in them whose direction, as found by Fleming’s Right-hand Rule, is in opposition to the applied voltage (Fig. 29.5). Because of its opposing direction, it is referred to as counter e.m.f. or back e.m.f. $E_b$. The equivalent circuit of a motor is shown in Fig. 29.6. The rotating armature generating the back e.m.f. $E_b$ is like a battery of e.m.f. $E_b$ put across a supply mains of $V$ volts. Obviously, $V$ has to drive $I_a$ against the opposition of $E_b$. The power required to overcome this opposition is $E_b I_a$.

In the case of a cell, this power over an interval of time is converted into chemical energy, but in the present case, it is converted into mechanical energy.

It will be seen that $I_a = \frac{\text{Net voltage}}{\text{Resistance}} = \frac{V - V_b}{R_a}$

where $R_a$ is the resistance of the armature circuit. As pointed out above, $E_b = \Phi ZN \times (P/A)$ volt where $N$ is in r.p.s.

Back e.m.f. depends, among other factors, upon the armature speed. If speed is high, $E_b$ is large, hence armature current $I_a$, seen from the above equation, is small. If the speed is less, then $E_b$ is less, hence more current flows which develops motor torque (Art 29.7). So, we find that $E_b$ acts like a governor i.e., it makes a motor self-regulating so that it draws as much current as is just necessary.

### 29.4. Voltage Equation of a Motor

The voltage $V$ applied across the motor armature has to

(i) overcome the back e.m.f. $E_b$ and

(ii) supply the armature ohmic drop $I_a R_a$

\[ V = E_b + I_a R_a \]

This is known as voltage equation of a motor.

Now, multiplying both sides by $I_a^2$, we get

\[ V I_a = E_b I_a + I_a^2 R_a \]

As shown in Fig. 29.6,

- $V I_a$ = Electrical input to the armature
- $E_b I_a$ = Electrical equivalent of mechanical power developed in the armature
- $I_a^2 R_a$ = Cu loss in the armature

Hence, out of the armature input, some is wasted in $I_a^2 R$ loss and the rest is converted into mechanical power within the armature.

It may also be noted that motor efficiency is given by the ratio of power developed by the arma-
ture to its input \( E_b I_a / V I_a = E_b / V \). Obviously, higher the value of \( E_b \) as compared to \( V \), higher the motor efficiency.

### 29.5. Condition for Maximum Power

The gross mechanical power developed by a motor is \( P_m = V I_a - I_a^2 R_a \).

Differentiating both sides with respect to \( I_a \) and equating the result to zero, we get

\[
d P_m / d I_a = V - 2 I_a R_a = 0 : \quad I_a R_a = V / 2
\]

Thus gross mechanical power developed by a motor is maximum when back e.m.f. is equal to half the applied voltage. This condition is, however, not realized in practice, because in that case current would be much beyond the normal current of the motor. Moreover, half the input would be wasted in the form of heat and taking other losses (mechanical and magnetic) into consideration, the motor efficiency will be well below 50 percent.

**Example 29.1.** A 220-V d.c. machine has an armature resistance of 0.5 \( \Omega \). If the full-load armature current is 20 A, find the induced e.m.f. when the machine acts as (i) generator (ii) motor.

(Electrical Technology-I, Bombay Univ. 1987)

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**Solution.** As shown in Fig. 29.7, the d.c. machine is assumed to be shunt-connected. In each case, shunt current is considered negligible because its value is not given.

(a) As Generator [Fig. 29.7(a)]

\[
E_g = V + I_a R_a = 220 + 0.5 \times 20 = 230 \text{ V}
\]

(b) As Motor [Fig. 29.7(b)]

\[
E_b = V - I_a R_a = 220 - 0.5 \times 20 = 210 \text{ V}
\]

**Example 29.2.** A separately excited D.C. generator has armature circuit resistance of 0.1 ohm and the total brush-drop is 2 V. When running at 1000 r.p.m., it delivers a current of 100 A at 250 V to a load of constant resistance. If the generator speed drop to 700 r.p.m., with field-current unaltered, find the current delivered to load.

(AMIE, Electrical Machines, 2001)

**Solution.**

\[
R_L = 250 / 100 = 2.5 \text{ ohms.}
\]

\[
E_{g1} = 250 + (100 \times 0.1) + 2 = 262 \text{ V.}
\]

At 700 r.p.m., \( E_{g2} = 262 \times 700 / 1000 = 183.4 \text{ V} \).

If \( I_a \) is the new current, \( E_{g2} - 2 - (I_a \times 0.1) = 2.5 I_a \)

This gives \( I_a = 96.77 \text{ amp.} \)

**Extension to the Question:** With what load resistance will the current be 100 amp, at 700 r.p.m.?

**Solution.**

\[
E_{g2} - 2 - (I_a \times 0.1) = R_L \times I_a
\]

For \( I_a = 100 \text{ amp,} \) and \( E_{g2} = 183.4 \text{ V,} \) \( R_L = 1.714 \text{ ohms.} \)

**Example 29.3.** A 440-V, shunt motor has armature resistance of 0.8 \( \Omega \) and field resistance of 200 \( \Omega \). Determine the back e.m.f. when giving an output of 7.46 kW at 85 percent efficiency.

**Solution.**

Motor input power = \( 7.46 \times 10^3 / 0.85 \text{ W} \)
Motor input current = $7460/0.85 \times 440 = 19.95 \, \text{A} \; ; \; I_{sh} = 440/200 = 2.2 \, \text{A}$

$I_a = 19.95 - 2.2 = 17.75 \, \text{A}$ \; ; \; Now, $E_b = V - I_a R_a$

$E_b = 440 - (17.75 \times 0.8) = 425.8 \, \text{V}$

**Example 29.4.** A 25-kW, 250-V, d.c. shunt generator has armature and field resistances of 0.06 $\Omega$ and 100 $\Omega$ respectively. Determine the total armature power developed when working (i) as a generator delivering 25 kW output and (ii) as a motor taking 25 kW input.

*(Electrical Technology, Punjab Univ., June 1991)*

**Solution. As Generator** [Fig. 29.8 (a)]

Output current = $25,000/250 = 100 \, \text{A} \; ; \; I_{sh} = 250/100 = 2.5 \, \text{A} \; ; \; I_a = 102.5 \, \text{A}$

Generated e.m.f. = $250 + I_a R_a = 250 + 102.5 \times 0.06 = 256.15 \, \text{V}$

Power developed in armature = $E_b I_a = \frac{256.15 \times 102.5}{1000} = 26.25 \, \text{kW}$

**As Motor** [Fig 29.8 (b)]

Motor input current = $100 \, \text{A} \; ; \; I_{sh} = 2.5 \, \text{A}, I_a = 97.5 \, \text{A}$

$E_b = 250 - (97.5 \times 0.06) = 250 - 5.85 = 244.15 \, \text{V}$

Power developed in armature = $E_b I_a = 244.15 \times 97.5/1000 = 23.8 \, \text{kW}$

**Example 29.5.** A 4 pole, 32 conductor, lap-wound d.c. shunt generator with terminal voltage of 200 volts delivering 12 amps to the load has $r_a = 2$ and field circuit resistance of 200 ohms. It is driven at 1000 r.p.m. Calculate the flux per pole in the machine. If the machine has to be run as a motor with the same terminal voltage and drawing 5 amps from the mains, maintaining the same magnetic field, find the speed of the machine.

*Sambalpur University, 1998*

**Solution.** Current distributions during two actions are indicated in Fig. 29.9 (a) and (b). As a generator, $I_a = 13 \, \text{amp}$

\[E_g = 200 + 13 \times 2 = 226 \, \text{V}\]
\[ \phi \frac{ZN}{60} \times \frac{P}{a} = 226 \]

For a Lap-wound armature,
\[ P = a \]
\[ \therefore \quad \phi = \frac{226 \times 60}{1000 \times 32} = 0.42375 \text{ wb} \]

As a motor,
\[ I_a = 4 \text{ amp} \]
\[ E_b = 200 - 4 \times 2 = 192 \text{ V} \]
\[ = \phi \frac{ZN}{60} \]

Giving \( N = \frac{60 \times 192}{0.42375 \times 32} \)
\[ = 850 \text{ r.p.m.} \]

**Tutorial Problems 29.1**

1. What do you understand by the term 'back e.m.f.'? A d.c. motor connected to a 460-V supply has an armature resistance of 0.15 \( \Omega \). Calculate
   
   (a) The value of back e.m.f. when the armature current is 120 A.
   
   (b) The value of armature current when the back e.m.f. is 447.4 V. \([a] 442 \text{ V} \quad [b] 84 \text{ A}]\)

2. A d.c. motor connected to a 460-V supply takes an armature current of 120 A on full load. If the armature circuit has a resistance of 0.25 \( \Omega \), calculate the value of the back e.m.f. at this load. \([430 \text{ V}]\)

3. A 4-pole d.c. motor takes an armature current of 150 A at 440 V. If its armature circuit has a resistance of 0.15 \( \Omega \), what will be the value of back e.m.f. at this load? \([417.5 \text{ V}]\)

**29.6. Torque**

By the term torque is meant the turning or twisting moment of a force about an axis. It is measured by the product of the force and the radius at which this force acts.

Consider a pulley of radius \( r \) metre acted upon by a circumferential force of \( F \) Newton which causes it to rotate at \( N \) r.p.m. (Fig. 29.10).

Then torque \( T = F \times r \text{ Newton-metre (N - m)} \)

Work done by this force in one revolution
\[ = \text{Force} \times \text{distance} = F \times 2\pi r \text{ Joule} \]

Power developed \( = F \times 2\pi r \times N \text{ Joule/second or Watt} \)
\[ = (F \times r) \times 2\pi N \text{ Watt} \]

Now \( 2\pi N = \text{Angular velocity} \omega \) in radian/second and \( F \times r = \text{Torque} \( T \)
\[ \therefore \quad \text{Power developed} = T \times \omega \text{ watt or } P = T \omega \text{ Watt} \]

Moreover, if \( N \) is in r.p.m., then
\[ \omega = 2\pi N/60 \text{ rad/s} \]
\[ \therefore \quad P = \frac{2\pi N}{60} \times T \quad \text{or} \quad P = \frac{2\pi}{60} \cdot NT = \frac{NT}{9.55} \]

**29.7. Armature Torque of a Motor**

Let \( T_a \) be the torque developed by the armature of a motor running at \( N \) r.p.m. If \( T_a \) is in \( N/M \), then power developed \( = T_a \times 2\pi N \text{ watt} \)

...\( \text{(i)} \)
We also know that electrical power converted into mechanical power in the armature (Art 29.4)

\[ E_b I_a \text{ watt} \]  

Equating (i) and (ii), we get \( T_a \times 2\pi N = E_b I_a \) \( \text{watt} \) (iii)

Since \( E_b = \Phi ZN \times (P/A) \text{ volt} \), we have

\[ T_a \times 2\pi N = \Phi ZN \left( \frac{P}{A} \right) \cdot I_a \text{ or } T_a = \frac{1}{2\pi} \cdot \Phi ZI_0 \left( \frac{P}{A} \right) N \cdot m \]

\[ = 0.159 \text{ N newton metre} \]

\[ T_a = 0.159 \Phi ZI_a \times (P/A) N \cdot m \]

\[ \therefore \]

**Note.** From the above equation for the torque, we find that \( T_a \propto \Phi I_a^2 \)

(a) In the case of a series motor, \( \Phi \) is directly proportional to \( I_a \) (before saturation) because field windings carry full armature current

\[ \therefore T_a \propto I_a^2 \]

(b) For shunt motors, \( \Phi \) is practically constant, hence \( T_a \propto I_a \)

As seen from (iii) above

\[ T_a = \frac{E_b I_a}{2\pi N} \text{ N - m - N in r.p.s.} \]

If \( N \) is in r.p.m., then

\[ T_a = \frac{E_b I_a}{2\pi N / 60} = \frac{60 E_b I_a}{2\pi N} = \frac{60 E_b I_a}{2\pi N} = 9.55 \frac{E_b I_a}{N} \text{ N-m} \]

**29.8. Shaft Torque \((T_{sh})\)**

The whole of the armature torque, as calculated above, is not available for doing useful work, because a certain percentage of it is required for supplying iron and friction losses in the motor.

The torque which is available for doing useful work is known as shaft torque \( T_{sh} \). It is so called because it is available at the shaft. The motor output is given by

Output = \( T_{sh} \times 2\pi N \text{ Watt provided } T_{sh} \text{ is in N-m and } N \text{ in r.p.s.} \)

\[ \therefore T_{sh} = \frac{\text{Output in watts}}{2\pi N} \text{ N - m - N in r.p.s} \]

\[ = \frac{\text{Output in watts}}{2\pi N / 60} \text{ N-m - N in r.p.m.} \]

\[ = \frac{60 \text{ output}}{2\pi} = 9.55 \frac{\text{Output}}{N} \text{ N-m.} \]

The difference \( (T_a - T_{sh}) \) is known as lost torque and is due to iron and friction losses of the motor.

**Note.** The value of back e.m.f. \( E_b \) can be found from

(i) the equation, \( E_b = V - I_a R_a \)

(ii) the formula \( E_b = \Phi ZN \times (P/A) \text{ volt} \)

**Example 29.6.** A d.c. motor takes an armature current of 110 A at 480 V. The armature circuit resistance is 0.2 \( \Omega \). The machine has 6-poles and the armature is lap-connected with 864 conductors. The flux per pole is 0.05 Wb. Calculate (i), the speed and (ii) the gross torque developed by the armature.

( *Elect. Machines, A.M.I.E. See B, 1989*)

**Solution.** \( E_b = 480 - 110 \times 0.2 = 458 \text{ V, } \Phi = 0.05 \text{ W, } Z = 864 \)

Now,

\[ E_b = \frac{\Phi ZN}{60} \left( \frac{P}{A} \right) \text{ or } 458 = \frac{0.05 \times 864 \times N}{60} \times \left( \frac{6}{6} \right) \]

\[ \therefore N = 636 \text{ r.p.m.} \]

\[ T_a = 0.159 \times 0.05 \times 864 \times 110 \left( \frac{6}{6} \right) = 756.3 \text{ N-m} \]
Example 29.7. A 250-V, 4-pole, wave-wound d.c. series motor has 782 conductors on its armature. It has armature and series field resistance of 0.75 ohm. The motor takes a current of 40 A. Estimate its speed and gross torque developed if it has a flux per pole of 25 mWb.

(Elect. Engg.-II, Pune Univ. 1991)

Solution.
\[ E_b = \Phi ZN (P/A) \]
Now,
\[ E_b = V - I_a R_a = 50 - 40 \times 0.75 = 220 \text{ V} \]
\[ \therefore \quad 220 = 25 \times 10^{-3} \times 782 \times N \times 0.75 = 220 \text{ V} \]
\[ \therefore \quad 220 = 0.159 \Phi Z I_a (P/A) \]
\[ = 0.159 \times 25 \times 10^{-3} \times 782 \times 40 \times (4/2) = 249 \text{ N-m} \]

Example 29.8. A d.c. shunt machine develops an a.c. e.m.f. of 250 V at 1500 r.p.m. Find its torque and mechanical power developed for an armature current of 50 A. State the simplifying assumptions.

(Basic Elect. Machine Nagpur Univ., 1993)

Solution. A given d.c. machine develops the same e.m.f. in its armature conductors whether running as a generator or as a motor. Only difference is that this armature e.m.f. is known as back e.m.f. when the machine is running as a motor.

Mechanical power developed in the arm = \( E_a I_a = 250 \times 50 = 12,500 \text{ W} \)
\[ T_a = 9.55 E_a I_a / N = 9.55 \times 250 \times 50 / 1500 = 79.6 \text{ N-m} \]

Example 29.9. Determine developed torque and shaft torque of 220-V, 4-pole series motor with 800 conductors wave-connected supplying a load of 8.2 kW by taking 45 A from the mains. The flux per pole is 25 mWb and its armature circuit resistance is 0.6 Ω.


Solution. Developed torque or gross torque is the same thing as armature torque.
\[ T_a = 0.159 \Phi Z A (P/A) \]
\[ = 0.159 \times 25 \times 10^{-3} \times 800 \times 45 (4/2) = 286.2 \text{ N-m} \]
\[ E_b = V - I_a R_a = 220 - 45 \times 0.6 = 193 \text{ V} \]
Now,
\[ E_b = 0.159 \Phi Z (P/A) \text{ or } 193 = 25 \times 10^{-3} \times 800 \times N \pi \times (4/2) \]
\[ \therefore \quad N = 4.825 \text{ r.p.s.} \]
Also,
\[ 2\pi N T_{sh} = \text{output or } 2\pi \times 4.825 T_{sh} = 8200 \quad \therefore \quad T_{sh} = 270.5 \text{ N-m} \]

Example 29.10. A 220-V d.c. shunt motor runs at 500 r.p.m. when the armature current is 50 A. Calculate the speed if the torque is doubled. Given that \( R_a = 0.2 \Omega \).

(Electrical Technology-II, Gwalior Univ. 1985)

Solution. As seen from Art 27.7, \( T_a \propto \Phi I_a \). Since \( \Phi \) is constant, \( T_a \propto I_a \).
\[ \therefore \quad T_{a1} \propto I_{a1} \quad \text{and} \quad T_{a2} \propto I_{a2} \quad \therefore \quad T_{a2}/T_{a1} = I_{a2}/I_{a1} \]
\[ \therefore \quad 2 = I_{a2}/50 \text{ or } I_{a2} = 100 \text{ A} \]
Now, \( N_1/N_2 = E_{b2}/E_{b1} \)
\[ F_{b1} = 220 - (50 \times 0.2) = 210 \text{ V} \]
\[ E_{b2} = 220 - (100 \times 0.2) = 200 \text{ V} \]
\[ \therefore \quad N_2/500 = 200/210 \quad \therefore \quad N_2 = 476 \text{ r.p.m.} \]

Example 29.11. A 500-V, 37.3 kW, 1000 r.p.m. d.c. shunt motor has on full-load an efficiency of 90 percent. The armature circuit resistance is 0.24 Ω and there is total voltage drop of 2 V at the brushes. The field current is 1.8 A. Determine (i) full-load line current (ii) full load shaft torque in N-m and (iii) total resistance in motor starter to limit the starting current to 1.5 times the full-load current.

(Elect. Engg. I; M.S. Univ. Baroda 1987)

Solution. (i) \( \text{Motor input} = 37,300/0.9 = 41,444 \text{ W} \)
\[ \text{F.L. line current} = 41,444/500 = 82.9 \text{ A} \]
(iii) \[ T_{sh} = 9.55 \frac{\text{output}}{N} = 9.55 \times \frac{37,300}{1000} = 356 \text{ N-m} \]

Starting line current \( = 1.5 \times 82.9 = 124.3 \text{ A} \)

Arm. current at starting \( = 124.3 - 1.8 = 122.5 \text{ A} \)

If \( R \) is the starter resistance (which is in series with armature), then

\[ 122.5 (R + 0.24) + 2 = 500 \quad \therefore \quad R = 3.825 \Omega \]

Example 29.12. A 4-pole, 220-V shunt motor has 540 lap-wound conductor. It takes 32 A from the supply mains and develops output power of 5.595 kW. The field winding takes 1 A. The armature resistance is 0.09 \( \Omega \) and the flux per pole is 30 mWb. Calculate (i) the speed and (ii) the torque developed in newton-metre.

(Electrical Technology, Nagpur Univ. 1992)

Solution. \( I_a = 32 - 1 = 31 \text{ A; } E_b = V - I_a R_a = 220 - (0.09 \times 31) = 217.2 \text{ V} \)

Now,

\[ E_b = \frac{\Phi Z N}{60} \left( \frac{P}{A} \right) \quad \therefore \quad 217.2 = \frac{30 \times 10^{-3} \times 540 \times N}{60} \left( \frac{4}{4} \right) \]

(i) \quad \therefore \quad N = 804.4 \text{ r.p.m.} \]

(ii) \[ T_{sh} = 9.55 \times \frac{\text{output in watts}}{N} = 9.55 \times \frac{5.595}{804.4} = 66.5 \text{ N-m} \]

Example 29.13 (a). Find the load and full-load speeds for a four-pole, 220-V, and 20-kW, shunt motor having the following data:

Field-current = 5 amp, armature resistance = 0.04 ohm,

Flux per pole = 0.04 Wb, number of armature-conductors = 160, Two-circuit wave-connection, full load current = 95 amp, No load current =9 A. Neglect armature reaction.

(Bharathithasan Univ. April 1997)

Solution. The machine draws a supply current of 9 amp at no load. Out of this, 5 amps are required for the field circuit, hence the armature carries a no-load current of 4 amp.

At load, armature-current is 90 amp. The armature-resistance-drop increases and the back e.m.f. decreases, resulting into decrease in speed under load compared to that at No-Load.

At No Load: \( E_{no} = 220 - 4 \times 0.04 = 219.84 \text{ volts} \)

Substituting this,

\[ 0.04 \times 160 \times (N/60) \times (4/2) = 219.84 \]

No-Load speed, \( N_0 = 1030.5 \text{ r.p.m.} \)

At Full Load: Armature current = 90 A, \( E_a = 200 - 90 \times 0.04 = 216.4 \text{ V} \)

\[ N = (216.4/219.84) \times 1030.5 = 1014.4 \text{ r.p.m.} \]

Example 29.13 (b). Armature of a 6-pole, 6-circuit D.C. shunt motor takes 400 A at a speed of 350 r.p.m. The flux per pole is 80 milli-webers, the number of armature turns is 600, and 3% of the torque is lost in windage, friction and iron-loss. Calculate the brake-horse-power.

(Manonmaniam Sundaranar Univ. Nov. 1998)

Solution. Number of armature turns = 600

Therefore, \( Z = \text{Number of armature conductors} = 1200 \)

If electromagnetic torque developed is \( T \text{ Nw} - \text{m} \),

Armature power = \( T \omega = T \times 2 \pi \times 350/60 \)

= 36.67 \( T \text{ watts} \)

To calculate armature power in terms of Electrical parameters, \( E \) must be known.

\[ E = \phi Z (N/60) (P/A) \]
\[
= 80 \times 10^{-3} \times 1200 \times (350/60) \times (6/6) \\
= 560 \text{ volts}
\]

With the armature current of 400 A, Armature power = 560 \times 400 \text{ watts}

Equating the two,
\[
T = 560 \times 400/36.67 = 6108.5 \text{ Nw} - \text{m. Since 3 \% of this torque is required for overcoming different loss-terms,}
\]
\[
\text{Net torque} = 0.97 \times 6180.5 = 5925 \text{ Nw} - \text{m}
\]

For Brake-Horse-Power, net output in kW should be computed first. Then “kW” is to be converted to “BHP”, with 1 HP = 0.746 kW.

Net output in kW = 5925 \times 36.67 \times 10^{-3} = 217.27 kW

Converting this to BHP, the output = 291.25 HP

**Example 29.13 (c).** Determine the torque established by the armature of a four-pole D.C. motor having 774 conductors, two paths in parallel, 24 milli-webers of pole-flux and the armature current is 50 Amps.

**(Bharathiar Univ. April 1998)**

**Solution.** Expression for torque in terms of the parameters concerned in this problem is as follows:

\[
T = 0.159 \phi Z I_a p/\alpha \text{ Nw} - \text{m}
\]

Two paths in parallel for a 4-pole case means a wave winding.
\[
T = 0.159 \times (24 \times 10^{-3}) \times 774 \times 50 \times 4/2 \\
= 295.36 \text{ Nw-m}
\]

**Example 29.13 (d).** A 500-V D.C. shunt motor draws a line-current of 5 A on light-load. If armature resistance is 0.15 ohm and field resistance is 200 ohms, determine the efficiency of the machine running as a generator delivering a load current of 40 Amps.

**(Bharathiar Univ. April 1998)**

**Solution.** (i) No Load, running as a motor:

\[
\text{Input Power} = 500 \times 5 = 2500 \text{ watts}
\]
\[
\text{Field copper-loss} = 500 \times 2.5 = 1250 \text{ watts}
\]

Neglecting armature copper-loss at no load (since it comes out to be \(2.5^2 \times 0.15 = 1 \text{ watt}\)), the balance of 1250 watts of power goes towards no load losses of the machine running at rated speed. These losses are mainly the no load mechanical losses and the core-loss.

(ii) As a Generator, delivering 40 A to load:

\[
\text{Output delivered} = 500 \times 40 \times 10^{-3} = 20 \text{ kW}
\]

Losses: (a) Field copper-loss = 1250 watts

(b) Armature copper-loss = \(42.5^2 \times 0.15 = 271 \text{ watts}\)

(c) No load losses = 1250 watts

Total losses = 2771 kW

Generator Efficiency = \((20/22.771) \times 100 \% = 87.83 \%\)

**Extension to the Question:** At what speed should the Generator be run, if the shunt-field is not changed, in the above case? Assume that the motor was running at 600 r.p.m. Neglect armature reaction.

**Solution.** As a motor on no-load,

\[
E_{b0} = 500 - I_a r_a = 500 - 0.15 \times 2.5 = 499.625 \text{ V}
\]

As a Generator with an armature current of 42.5 A,
\[ E_{bo} = 500 + 42.5 \times 0.15 = 506.375 \text{ V} \]

Since, the terminal voltage is same in both the cases, shunt field current remains as 2.5 amp.
With armature reaction is ignored, the flux/pole remains same. The e.m.f. then becomes proportional to the speed. If the generator must be driven at \( N \) r.p.m.

\[ N = \left( \frac{506.375}{449.625} \right) \times 600 = 608.1 \text{ r.p.m.} \]

(a) Motor at no load

(b) Generator loaded

Fig. 29.11

Note. Alternative to this slight increase in the speed is to increase the field current with the help of decreasing the rheostatic resistance in the field-circuit.

Example 29.13 (e). A d.c. series motor takes 40 A at 220 V and runs at 800 r.p.m. If the armature and field resistance are 0.2 \( \Omega \) and 0.1 \( \Omega \) respectively and the iron and friction losses are 0.5 kW, find the torque developed in the armature. What will be the output of the motor ?

Solution. Armature torque is given by \( T_a = 9.55 \frac{E_b \cdot I_a}{N} \text{ N-m} \)

Now

\[ E_b = V - I_a (R_a + R_{se}) = 220 - 40 (0.2 + 0.1) = 208 \text{ V} \]

\[ \therefore \quad T_a = 9.55 \times 208 \times 40/800 = 99.3 \text{ N-m} \]

Cu loss in armature and series-field resistance = \( 40^2 \times 0.3 = 480 \text{ W} \)

Iron and friction losses = 500 W ; Total losses = 480 + 500 = 980 W

Motor power input = 220 \times 40 = 8,800 \text{ W} 

Motor output = 8,800 - 980 = 7,820 \text{ W} = 7.82 \text{ kW} 

Example 29.14. A cutting tool exerts a tangential force of 400 N on a steel bar of diameter 10 cm which is being turned in a simple lathe. The lathe is driven by a chain at 840 r.p.m. from a 220 V d.c. Motor which runs at 1800 r.p.m. Calculate the current taken by the motor if its efficiency is 80 \%. What size is the motor pulley if the lathe pulley has a diameter of 24 cm?

(Elect. Technology-II, Gwalior Univ. 1985)

Solution. Torque \( T_{sh} = \text{Tangential force \times radius} = 400 \times 0.05 = 20 \text{ N-m} \)

Output power = \( T_{sh} \times 2\pi N \text{ watt} = 20 \times 2\pi \times (840/60) \text{ watt} = 1,760 \text{ W} \)

Motor \( \eta = 0.8 \quad \therefore \quad \text{Motor input} = 1,760/0.8 = 2,200 \text{ W} \)

Current drawn by motor = \( 2200/220 = 10 \text{ A} \)

Let \( N_1 \) and \( D_1 \) be the speed and diameter of the driver pulley respectively and \( N_2 \) and \( D_2 \) the respective speed and diameter of the lathe pulley.

Then

\[ N_1 \times D_1 = N_2 \times D_2 \quad \text{or} \quad 1,800 \times D_1 = 840 \times 0.24 \]

\[ \therefore \quad D_1 = \frac{840 \times 0.24}{1,800} = 0.112 \text{ m} = 11.2 \text{ cm} \]

Example 29.15. The armature winding of a 200-V, 4-pole, series motor is lap-connected. There are 280 slots and each slot has 4 conductors. The current is 45 A and the flux per pole is 18 mWb. The field resistance is 0.3 \( \Omega \); the armature resistance 0.5 \( \Omega \) and the iron and friction losses total 300 W. The pulley diameter is 0.41 m. Find the pull in newton at the rim of the pulley.

(Elect. Engg. AMIETE Sec. A. 1991)
Solution. 
\[ E_b = V - I_a R_a = 200 - 45 (0.5 + 0.3) = 164 \text{ V} \]

Now 
\[ E_b = \frac{\Phi ZN}{60} \cdot \left(\frac{P}{A}\right) \text{ volt} \]

\[ \therefore \quad 164 = \frac{18 \times 10^{-3} \times 280 \times 4 \times N}{60} \times \frac{4}{4} \quad \therefore \quad N = 488 \text{ r.p.m.} \]

Total input = \( 200 \times 45 = 9,000 \text{ W} \); Cu loss = \( I_a^2 R_a = 45^2 \times 0.8 = 1,620 \text{ W} \)

Iron + Friction losses = \( 800 \text{ W} \); Total losses = \( 1,620 + 800 = 2,420 \text{ W} \)

Output = \( 9,000 - 2,420 = 6,580 \text{ W} \)

\[ \therefore \quad T_{sh} = 9 \times 55 \times \frac{6580}{488} = 128 \text{ N-m} \]

Let \( F \) be the pull in newtons at the rim of the pulley.

Then 
\[ F \times 0.205 = 128.8 \quad \therefore \quad F = 128.8/0.205 \quad N = 634 \text{ N} \]

Example 29.16. A 4-pole, 240 V, wave connected shunt motor gives 1119 kW when running at 1000 r.p.m. and drawing armature and field currents of 50 A and 1.0 A respectively. It has 540 conductors. Its resistance is 0.1 \( \Omega \). Assuming a drop of 1 volt per brush, find (a) total torque (b) useful torque (c) useful flux/pole (d) rotational losses and (e) efficiency.

Solution. 
\[ E_b = V - I_a R_a - \text{ brush drop} = 240 - (50 \times 0.1) - 2 = 233 \text{ V} \]

Also 
\[ I_a = 50 \text{ A} \]

(a) 
Armature torque \( T_a = 9.55 \frac{E_b I_a}{N} \text{ N-m} = 9.55 \times \frac{233 \times 50}{1000} = 111 \text{ N-m} \)

(b) 
\[ T_{sh} = 9.55 \left( \frac{\text{output}}{N} \right) = 9.55 \times \frac{11,190}{1000} = 106.9 \text{ N-m} \]

(c) 
\[ E_b = \frac{\Phi ZN}{60} \times \left(\frac{P}{A}\right) \text{ volt} \]

\[ \therefore \quad 233 = \frac{\Phi \times 540 \times 1000}{60} \times \left(\frac{4}{2}\right) \quad \therefore \quad \Phi = 12.9 \text{ mWb} \]

(d) 
Armature input = \( V I_a = 240 \times 50 = 12,000 \text{ W} \)

Armature Cu loss = \( I_a^2 R_a = 50^2 \times 0.1 = 250 \text{ W} \); Brush contact loss = \( 50 \times 2 = 100 \text{ W} \)

\[ \therefore \quad \text{Power developed} = 12,000 - 350 = 11,650 \text{ W} \]; Output = 11.19 kW = 11,190 W

\[ \therefore \quad \text{Rotational losses} = 11,650 - 11,190 = 460 \text{ W} \]

(e) 
Total motor input = \( VI = 240 \times 51 = 12,340 \text{ W} \); Motor output = 11,190 W

\[ \therefore \quad \text{Efficiency} = \frac{11,190}{12,240} \times 100 = 91.4 \% \]

Example 29.17. A 460-V series motor runs at 500 r.p.m. taking a current of 40 A. Calculate the speed and percentage change in torque if the load is reduced so that the motor is taking 30 A. Total resistance of the armature and field circuits is 0.8 \( \Omega \). Assume flux is proportional to the field current.

(Elect. Engg.-II, Kerala Univ. 1988)

Solution. Since \( \Phi \propto I_a \), hence \( T \propto I_a^2 \)

\[ \therefore \quad T_1 \propto 40^2 \quad \text{and} \quad T_2 \propto 30^2 \quad \therefore \quad \frac{T_2}{T_1} = \frac{9}{16} \]

\[ \therefore \quad \% \text{Percentage change in torque is} \]

\[ \frac{T_1 - T_2}{T_1} \times 100 = \frac{7}{16} \times 100 = 43.75 \% \]

Now \( E_{b1} = 460 - (40 \times 0.8) = 428 \text{ V} \); \( E_{b2} = 460 - (30 \times 0.8) = 436 \text{ V} \)

\[ \frac{N_2}{N_1} = \frac{E_{b2} \times I_{a1}}{E_{b1} I_{a2}} \quad \therefore \quad \frac{N_2}{500} = \frac{436 \times 40}{428 \times 30} \quad \therefore \quad N_2 = 679 \text{ r.p.m.} \]
Example 29.18. A 460-V, 55.95 kW, 750 r.p.m. shunt motor drives a load having a moment of inertia of 252.8 kg m². Find approximate time to attain full speed when starting from rest against full-load torque if starting current varies between 1.4 and 1.8 times full-load current.

Solution. Let us suppose that the starting current has a steady value of \((1.4 + 1.8)/2 = 1.6\) times full-load value.

Full-load output \(= 55.95 \text{ kW} = 55,950 \text{ W}\); Speed = 750 r.p.m. = 12.5 r.p.s.

F.L. shaft torque \(T = \text{power}/\omega = \text{power}/2\pi N = 55,950 \pi \times (750/60) = 712.4 \text{ N-m}\)

During starting period, average available torque

\(= 1.6 T - T = 0.6 T = 0.6 \times 712.4 = 427.34 \text{ N-m}\)

This torque acts on the moment of inertia \(I = 252.8 \text{ km m}^2\).

\[
252.8 \times \frac{d\omega}{dt} = 252.8 \times \frac{2\pi \times 12.5}{dt}, \quad \therefore \quad dt = 46.4 \text{ s}
\]

Example 29.19. A 14.92 kW, 400 V, 400 r.p.m. d.c. shunt motor draws a current of 40 A when running at full-load. The moment of inertia of the rotating system is 7.5 kg m². If the starting current is 1.2 times full-load current, calculate

(a) full-load torque
(b) the time required for the motor to attain the rated speed against full-load.

(Electrical Technology, Gujarat Univ. 1988)

Solution. (a) F.L. output \(14.92 \text{ kW} = 14,920 \text{ W}\); Speed = 400 r.p.m. = 20/3 r.p.s

Now, \(T\omega = \text{output} \cdot T = 14,920/2\pi \times (20/3) = 356 \text{ N-m}\)

(b) During the starting period, the torque available for accelerating the motor armature is

\(= 1.2 T - T = 0.2 T = 0.2 \times 356 = 71.2 \text{ N-m}\)

Now, torque \(= I \frac{d\omega}{dt}\) \(\therefore 71.2 = 7.5 \times \frac{2\pi \times (20/3)}{dt} \therefore dt = 4.41 \text{ second}\)

29.9. Speed of a D.C. Motor

From the voltage equation of a motor (Art. 27.4), we get

\[E_b = V - I_a R_a \quad \text{or} \quad \frac{\Phi Z N}{60} \left(\frac{P}{A}\right) = V - I_a R_a\]

\[\therefore \quad N = \frac{V - I_a R_a}{\Phi} \times \left(\frac{60A}{ZP}\right) \text{ r.p.m.}\]

Now

\[V - I_a R_a = E_b \quad \therefore \quad N = \frac{E_b}{\Phi} \times \left(\frac{60A}{ZP}\right) \text{ r.p.m. or } N = K \frac{E_b}{\Phi}\]

It shows that speed is directly proportional to back e.m.f. \(E_b\) and inversely to the flux \(\Phi\) on \(N \propto E_b/\Phi\).

For Series Motor

Let

\[N_1 = \text{Speed in the 1st case; } I_{a1} = \text{armature current in the 1st case}\]

\[\Phi_1 = \text{flux/pole in the first case}\]

\[N_2, I_{a2}, \Phi_2 = \text{corresponding quantities in the 2nd case.}\]

Then, using the above relation, we get

\[N_1 \propto \frac{E_b}{\Phi_1} \quad \text{where } E_{b1} = V - I_{a1} R_a; \quad N_2 \propto \frac{E_{b2}}{\Phi_2} \quad \text{where } E_{b2} = V - I_{a2} R_a\]

\[\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}\]

Prior to saturation of magnetic poles; \(\Phi \propto I_a\) \(\therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}\]
For Shunt Motor

In this case the same equation applies,

\[ \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \]

If \( \Phi_2 = \Phi_1 \), then \( \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \).

29.10. Speed Regulation

The term speed regulation refers to the change in speed of a motor with change in applied load torque, other conditions remaining constant. By change in speed here is meant the change which occurs under these conditions due to inherent properties of the motor itself and not those changes which are affected through manipulation of rheostats or other speed-controlling devices.

The speed regulation is defined as the change in speed when the load on the motor is reduced from rated value to zero, expressed as percent of the rated load speed.

\[ \text{\% speed regulation} = \frac{\text{N.L. speed} - \text{F.L. speed}}{\text{F.L. speed}} \times 100 = \frac{dN}{N} \times 100 \]

29.11. Torque and Speed of a D.C. Motor

It will be proved that though torque of a motor is admittedly a function of flux and armature current, yet it is independent of speed. In fact, it is the speed which depends on torque and not vice-versa. It has been proved earlier that

\[ N = K \frac{V - I_a R_a}{\Phi} = \frac{K E_{b}}{\Phi} \] ...Art. 27.9

Also,

\[ T_a \propto \Phi I_a \] ...Art. 27.7

It is seen from above that increase in flux would decrease the speed but increase the armature torque. It cannot be so because torque always tends to produce rotation. If torque increases, motor speed must increase rather than decrease. The apparent inconsistency between the above two equations can be reconciled in the following way:

Suppose that the flux of a motor is decreased by decreasing the field current. Then, following sequence of events take place:

1. Back e.m.f. \( E_b (= N \Phi/K) \) drops instantly (the speed remains constant because of inertia of the heavy armature).
2. Due to decrease in \( E_b \), \( I_a \) is increased because \( I_a = (V - E_b)/R_a \). Moreover, a small reduction in flux produces a proportionately large increase in armature current.
3. Hence, the equation \( T_a \propto \Phi I_a \), a small decrease in \( \Phi \) is more than counterbalanced by a large increase in \( I_a \) with the result that there is a net increase in \( T_a \).
4. This increase in \( T_a \) produces an increase in motor speed.

It is seen from above that with the applied voltage \( V \) held constant, motor speed varies inversely as the flux. However, it is possible to increase flux and, at the same time, increase the speed provided \( I_a \) is held constant as is actually done in a d.c. servomotor.

**Example 29.20.** A 4-pole series motor has 944 wave-connected armature conductors. At a certain load, the flux per pole is 34.6 mWb and the total mechanical torque developed is 209 N·m. Calculate the line current taken by the motor and the speed at which it will run with an applied voltage of 500 V. Total motor resistance is 3 ohm.

(Elect. Engg. AMIETE Sec. A Part II June 1991)

**Solution.**

\[ T_a = 0.159 \phi Z I_a (P/A) \text{ N·m} \]

\[ 209 = 0.159 \times 34.6 \times 10^{-3} \times 944 \times I_a (4/2); I_a = 20.1 \text{ A} \]

\[ E_a = V - I_a R_a = 500 - 20.1 \times 3 = 439.7 \text{ V} \]
Now, speed may be found either by using the relation for $E_b$ or $T_a$ as given in Art. 

\[ E_b = \Phi ZN \times (P/A) \text{ or } 439.7 = 34.6 \times 10^{-3} \times 944 \times N \times 2 \]

\[ N = 6.73 \text{ r.p.s. or } 382.2 \text{ r.p.m.} \]

Example 29.21. A 250-V shunt motor runs at 1000 r.p.m. at no-load and takes 8A. The total armature and shunt field resistances are respectively 0.2 $\Omega$ and 250 $\Omega$. Calculate the speed when loaded and taking 50 A. Assume the flux to be constant.  


Solution. Formula used: 

\[ \frac{N}{N_0} = \frac{E_{b0}}{E_{b0}} \times \frac{\Phi_0}{\Phi} \]

Since $\Phi_0 = \Phi$ (given), 

\[ \frac{N}{N_0} = \frac{E_{b0}}{E_{b0}} \]

\[ I_{sh} = \frac{250}{250} = 1 \text{ A} \]

\[ E_{b0} = V - I_{a0} R_a = 250 - (7 \times 0.2) = 248.6 \text{ V} \]

\[ E_b = V - I_a R_a = 250 - (49 \times 0.2) = 240.2 \text{ V} \]

\[ \frac{N}{1000} = \frac{240.2}{248.6} ; \quad N = 9666.1 \text{ r.p.m.} \]

Example 29.22. A d.c. series motor operates at 800 r.p.m. with a line current of 100 A from 230-V mains. Its armature circuit resistance is 0.15 $\Omega$ and its field resistance 0.1 $\Omega$. Find the speed at which the motor runs at a line current of 25 A, assuming that the flux at this current is 45 per cent of the flux at 100 A. 

(Electrical Machinery - I, Banglore Univ. 1986)

Solution.

\[ \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \]

\[ \Phi_2 = 0.45 \Phi_1 \text{ or } \frac{\Phi_1}{\Phi_2} = \frac{1}{0.45} \]

\[ E_{b1} = 230 - (0.15 + 0.1) \times 100 = 205 \text{ V} \]

\[ E_{b2} = 230 - 25 \times 0.25 = 223.75 \text{ V} \]

\[ \frac{N_2}{800} = \frac{223.75}{205} \times \frac{1}{0.45} ; \quad N_2 = 1940 \text{ r.p.m.} \]

Example 29.23. A 230-V d.c. shunt motor has an armature resistance of 0.5 $\Omega$ and field resistance of 115 $\Omega$. At no load, the speed is 1,200 r.p.m. and the armature current 2.5 A. On application of rated load, the speed drops to 1,120 r.p.m. Determine the line current and power input when the motor delivers rated load. 

(Elect. Technology, Kerala Univ. 1988)

Solution.

\[ N_1 = 1200 \text{ r.p.m., } E_{b1} = 230 - (0.5 \times 2.5) = 228.75 \text{ V} \]

\[ N_2 = 1120 \text{ r.p.m., } E_{b2} = 230 - 0.5 I_{a2} \]

Now,

\[ \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \quad \frac{1120}{1200} = \frac{230 - 0.5 I_{a2}}{228.75} ; \quad I_{a2} = 33 \text{ A} \]

Line current drawn by motor

\[ I_{a0} + I_{sh} = 33 + (230/115) = 35 \text{ A} \]

Power input at rated load

\[ 230 \times 35 = 8050 \text{ W} \]

Example 29.24. A belt-driven 100-kW, shunt generator running at 300 r.p.m. on 220-V busbars continues to run as a motor when the belt breaks, then taking 10 kW. What will be its speed? Given armature resistance = 0.025 $\Omega$, field resistance = 60 $\Omega$ and contact drop under each brush = 1 V. Ignore armature reaction. 

(Elect. Machines (E-3) AMIE Sec.C Winter 1991)

Solution. As Generator [Fig. 29.12 (a)]

Load current,

\[ I = 100,000/220 = 454.55 \text{ A} ; \quad I_{sh} = 220/60 = 3.67 \text{ A} \]

\[ I_a = I + I_{sh} = 458.2 \text{ A} ; \quad I_a R_a = 458.2 \times 0.025 = 11.45 \]

\[ E_b = 220 + 11.45 + 2 \times 1 = 233.45 \text{ V} ; \quad N_i = 300 \text{ r.p.m.} \]
As Motor [Fig. 29.12 (b)]

Input line current = 100,000/220 = 45.45 A; \( I_{sh} = 220/60 = 3.67 \) A

\[
I_a = 45.45 - 3.67 = 41.78 \text{ A}; \quad I_a R_a = 41.78 \times 0.025 = 1.04 \text{ V}; \quad E_{b2} = 220 - 1.04 - 2 \times 1 = 216.96 \text{ V}
\]

\[
\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}; \quad \text{since } \Phi_1 = \Phi_2 \text{ because } I_{sh} \text{ is constant}
\]

\[
\frac{N_2}{300} = \frac{216.96}{233.45}; \quad N_2 = 279 \text{ r.p.m.}
\]

Example 29.25. A d.c. shunt machine generates 250-V on open circuit at 1000 r.p.m. Effective armature resistance is 0.5 Ω, field resistance is 250 Ω, input to machine running as a motor on no-load is 4 A at 250 V. Calculate speed of machine as a motor taking 40 A at 250 V. Armature reaction weakens field by 4%.

(Electrical Machines-I, Gujarat Univ. 1987)

Solution. Consider the case when the machine runs as a motor on no-load.

Now, \( I_{sh} = 250/250 = 1 \) A; Hence, \( I_{a0} = 4 - 1 = 3 \) A; \( E_{b0} = 250 - 0.5 \times 3 = 248.5 \) V

It is given that when armature runs at 1000 r.p.m., it generates 250 V. When it generates 248.5 V, it must be running at a speed = \( 1000 \times 248.5/250 = 994 \) r.p.m.

Hence, \( N_0 = 994 \text{ r.p.m.} \)

When Loaded

\[
I_a = 40 - 1 = 39 \text{ A}; \quad E_b = 250 - 39 \times 0.5 = 230.5 \text{ V}
\]

Also, \( \Phi_0/\Phi = 1/0.96 \)

\[
\frac{N}{E} = \frac{E_b}{E_{b0}}; \quad \frac{N}{994} = \frac{230.5}{248.5} \times \frac{1}{0.96}
\]

\( N = 960 \text{ r.p.m.} \)

Example 29.26. A 250-V shunt motor giving 14.92 kW at 1000 r.p.m. takes an armature current of 75 A. The armature resistance is 0.25 ohm and the load torque remains constant. If the flux is reduced by 20 percent of its normal value before the speed changes, find the instantaneous value of the armature current and the torque. Determine the final value of the armature current and speed.

(Elect. Engg. AMIETE (New Scheme) 1990)

Solution. \( E_{b1} = 250 - 75 \times 0.25 = 231.25 \text{ V}, \) as in Fig. 29.13.

When flux is reduced by 20%, the back e.m.f. is also reduced instantly by 20% because speed remains constant due to inertia of the heavy armature (Art. 29.11).

\[
\Phi_0 = \Phi_{0.8} \quad \text{or} \quad E_{b1} = 0.8 E_{b2}
\]

\[
(I_a)_{\text{inst}} = \frac{[V - (E_{b1})_{\text{inst}}]}{R_a} = (250 - 185)/0.25 = 260 \text{ A}
\]

Fig. 29.13
Instantaneous value of the torque \( T_{a} \) inst = \( 9.55 \times \frac{(E_b)_{\text{inst}} \times (I_a)_{\text{inst}}}{N} \) (in r.p.m.)

or \( (T_a)_{\text{inst}} = 9.55 \times 185 \times 260/1000 = 459 \text{ N-m} \)

**Steady Conditions**

Since torque remains constant, \( \Phi_1 I_{a1} = \Phi_2 I_{a2} \)

\[ I_{a2} = \frac{\Phi_1}{\Phi_2} I_{a1} = 75 \times \frac{\Phi_1}{0.8} = 93.7 \text{ A} \]

\[ E_{b2} = \Phi_1 I_{a1} - E_{b1} = 250 - 93.7 \times 0.25 = 226.6 \text{ V} \]

Now,

\[ \frac{N_1}{N_2} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{226.6}{231.25} \times \frac{1}{0.8} ; N_2 = 1225 \text{ r.p.m.} \]

**Example 29.27.** A 220-V, d.c. shunt motor takes 4 A at no-load when running at 700 r.p.m. The field resistance is 100 \( \Omega \). The resistance of armature at standstill gives a drop of 6 volts across armature terminals when 10 A were passed through it. Calculate (a) speed on load (b) torque in N-m and (c) efficiency. The normal input of the motor is 8 kW.

(Electrotechnics-II; M.S. Univ. Baroda 1988)

**(a)** \( I_{sh} = 200/100 = 2 \text{ A} \)

F.L. Power input = 8,000 W; F.L. line current = 8,000/200 = 40 A

\[ I_a = 40 - 2 = 38 \text{ A} ; \quad R_a = 6/10 = 0.6 \Omega \]

\[ E_{b0} = 200 - 2 \times 0.6 = 198.8 \text{ V} ; \quad E_b = 200 - 38 \times 0.6 = 177.2 \text{ V} \]

Now,

\[ \frac{N_0}{N_1} = \frac{E_b}{E_{b0}} \quad \text{or} \quad \frac{N}{700} = \frac{177.2}{198.8} ; N = 623.9 \text{ r.p.m.} \]

**(b)** \( T_a = 9.55 E_b I_a / N = 9.55 \times 177.2 \times 38/623.9 = 103 \text{ N-m} \)

**(c)** N.L. power input = 200 \times 4 = 800 W; Arm. Cu loss = \( I_a^2 R_a = 38^2 \times 0.6 = 2.4 \text{ W} \)

Constant losses = 800 - 2.4 = 797.6 W; F.L. arm. Cu loss = 38^2 \times 0.6 = 866.4 W

Total F.L. losses = 797.6 + 866.4 = 1664 W; F.L. output = 8,000 - 1664 = 6336 W

F.L. Motor efficiency = 6336/8,000 = 0.792 or 79.2 %

**Example 29.28.** The input to 230-V, d.c. shunt motor is 11kW. Calculate (a) the torque developed (b) the efficiency (c) the speed at this load. The particulars of the motor are as follows:

No-load current = 5 A; No-load speed = 1150 r.p.m.

Arm. resistance = 0.5 \( \Omega \); shunt field resistance = 110 \( \Omega \).

(Elect. Technology; Bombay University 1988)

**(a)** No-load input = 220 \times 5 = 1,100 W; \( I_{sh} = 220/110 = 2 \text{A}; I_{ao} = 5 - 2 = 3 \text{A} \)

\[ \text{No-load armature Cu loss} = 3^2 \times 0.5 = 4.5 \text{ W} \]

\[ \therefore \text{Constant losses} = 1,100 - 4.5 = 1,095.5 \text{ W} \]

When input is 11 kW.

Input current = 11,000/220 = 50A; Armature current = 50 - 2 = 48 A

Arm. Cu loss = \( 48^2 \times 0.5 = 1,152 \text{ W} \);

Total loss = Arm. Cu loss + Constant losses = 1152 + 1095.5 = 2248 W

Output = 11,000 - 2,248 = 8,752 W

**(b)** Efficiency = 8,752 \times 100/11,000 = 79.6 %

**(c)** Back e.m.f. at no-load = 220 - (3 \times 0.5) = 218.5 V

Back e.m.f. at given load = 220 - (48 \times 0.5) = 196 V

\[ \therefore \text{Speed} N = 1,150 \times 196/218.5 = 1,031 \text{ r.p.m.} \]
Example 29.29. The armature circuit resistance of a 18.65 kW 250-V series motor is 0.1 Ω, the brush voltage drop is 3V, and the series field resistance is 0.05. When the motor takes 80 A, speed is 600 r.p.m. Calculate the speed when the current is 100 A.


Solution.

\[ E_{b1} = 250 - 80 (0.1 + 0.05) - 3 = 235 \text{ V} \]
\[ E_{b2} = 250 - 100 (0.1 + 0.05) - 3 = 232 \text{ V} \]

Since \( \Phi \propto I_a \), hence, \( \Phi_1 \propto 80, \Phi_2 \propto 100, \Phi_1/\Phi_2 = 80/100 \)

Now

\[ \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \quad \text{or} \quad \frac{N_2}{600} = \frac{232}{235} \times \frac{80}{100} \quad ; \quad N_2 = 474 \text{ r.p.m.} \]

Example 29.30. A 220-volt d.c. series motor is running at a speed of 800 r.p.m. and draws 100 A. Calculate at what speed the motor will run when developing half the torque. Total resistance of the armature and field is 0.1 ohm. Assume that the magnetic circuit is unsaturated.


Solution.

\[ \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} \quad (\because \Phi \propto I_a) \]

Since field is unsaturated, \( T_a \propto \Phi I_a \propto I_a^2 \) \( (\because T_1 \propto I_{a1}^2 \text{ and } T_2 \propto I_{a2}^2) \)

or

\[ \frac{T_2}{T_1} = \left( \frac{I_{a2}}{I_{a1}} \right)^2 \quad \text{or} \quad 1/2 = \left( \frac{I_{a2}}{I_{a1}} \right)^2 ; \quad I_{a1} = I_{a1}/\sqrt{2} = 70.7 \text{ A} \]

\[ E_{b1} = 220 - 100 \times 0.1 = 210 \text{ V} \quad ; \quad E_{b2} = 220 - 0.1 \times 70.7 = 212.9 \text{ V} \]

\[ \therefore \quad \frac{N_2}{800} = \frac{212.9}{210} \times \frac{100}{70.7} ; \quad N_2 = 1147 \text{ r.p.m.} \]

Example 29.31. A 4-pole d.c. motor runs at 600 r.p.m. on full load taking 25 A at 450 V. The armature is lap-wound with 500 conductors and flux per pole is expressed by the relation.

\[ \Phi = (1.7 \times 10^{-2} \times I^{0.5}) \text{ weber} \]

where \( I \) is the motor current. If supply voltage and torque are both halved, calculate the speed at which the motor will run. Ignore stray losses.

(Elect. Machines, Nagpur Univ. 1993)

Solution. Let us first find \( R_a \)

Now

\[ N = \frac{E_b}{Z\Phi} \left( \frac{60 \text{ A}}{P} \right) \text{ r.p.m.} \]

\[ \therefore \quad 600 = \frac{E_b}{1.7 \times 10^{-2} \times 25^{0.5} \times 500 \times 4} \]

\[ \therefore \quad E_b = 10 \times 1.7 \times 10^{-2} \times 5 \times 500 = 425 \text{ V} \]

\[ I_{a1} R_a = 450 - 425 = 25 \text{ V} ; \quad R_a = 25/25 = 1.0 \Omega \]

Now in the Ist Case

\[ T_1 \propto \Phi_1 I_1 \quad : \quad T_1 \propto 1.7 \times 10^{-2} \times 1 \times I \]

Similarly

\[ T_2 \propto 1.7 \times 10^{-2} \times \sqrt{I} \quad ; \quad \text{Now} \quad I_1 = 2T_2 \]

\[ \therefore \quad 1.7 \times 10^{-2} \times 125 = 1.7 \times 10^{-2} \times I/2^2 \times 2 \quad \therefore \quad I = (125/2)^{2/3} = 15.75 \text{ A} \]

\[ E_{b1} = 425 \text{ V} ; \quad E_{b2} = 225 - (15.75 \times 1) = 209.3 \text{ V} \]

Using the relation

\[ \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \quad ; \quad \text{we have} \]
\[
\frac{N_2}{600} = \frac{209.3}{425} \times \frac{1.7 \times 10^{-2} \times 5}{1.7 \times 10^{-2} \times \sqrt{15.75}}; \quad N_2 = 372 \text{ r.p.m.}
\]

**Tutorial Problems 29.2**

1. Calculate the torque in newton-metre developed by a 440-V d.c. motor having an armature resistance of 0.25 Ω and running at 750 r.p.m. when taking a current of 60 A. [325 N-m]

2. A 4-pole, lap-connected d.c. motor has 576 conductors and draws an armature current of 10 A. If the flux per pole is 0.02 Wb, calculate the armature torque developed. [18.3 N-m]

3. (a) A d.c. shunt machine has armature and field resistances of 0.025 Ω and 80 Ω respectively. When connected to constant 400-V bus-bars and driven as a generator at 450 r.p.m., it delivers 120 kW. Calculate its speed when running as a motor and absorbing 120 kW from the same bus-bars.

   (b) Deduce the direction of rotation of this machine when it is working as a motor assuming a clockwise rotation as a generator. [(a) 435 r.p.m. (b) Clockwise]

4. The armature current of a series motor is 60 A when on full-load. If the load is adjusted to that this current decreases to 40-A, find the new torque expressed as a percentage of the full-load torque. The flux for a current of 40 A is 70% of that when current is 60 A. [46%]

5. A 4-pole, d.c. shunt motor has a flux per pole of 0.04 Wb and the armature is lap-wound with 720 conductors. The shunt field resistance is 240 Ω and the armature resistance is 0.2 Ω. Brush contact drop is 1V per brush. Determine the speed of the machine when running (a) as a motor taking 60 A and (b) as a generator supplying 120 A. The terminal voltage in each case is 480 V. [972 r.p.m.; 1055 r.p.m.]

6. A 25-kW shunt generator is delivering full output to 400-V bus-bars and is driven at 950 r.p.m. by belt drive. The belt breaks suddenly but the machine continues to run as a motor taking 25 kW from the bus-bars. At what speed does it run? Take armature resistance including brush contact resistance as 0.5 Ω and field resistance as 160 Ω. [812.7 r.p.m.] (Elect. Technology, Andhra Univ. Apr. 1977)

7. A 4-pole, d.c. shunt motor has a wave-wound armature with 65 slots each containing 6 conductors. The flux per pole is 20 mWb and the armature has a resistance of 0.15 Ω. Calculate the motor speed when the machine is operating from a 250-V supply and taking a current of 60 A. [927 r.p.m.]

8. A 500-V, d.c. shunt motor has armature and field resistances of 0.5 Ω and 200 Ω respectively. When loaded and taking a total input of 25 kW, it runs at 400 r.p.m. Find the speed at which it must be driven as a shunt generator to supply a power output of 25 kW at a terminal voltage of 500 V. [442 r.p.m.]

9. A d.c. shunt motor runs at 900 r.p.m. from a 400 V supply when taking an armature current of 25 A. Calculate the speed at which it will run from a 230 V supply when taking an armature current of 15 A. The resistance of the armature circuit is 0.8 Ω. Assume the flux per pole at 230 V to have decreased to 75% of its value at 400 V. [595 r.p.m.]

10. A shunt machine connected to 250-A mains has an armature resistance of 0.12 Ω and field resistance of 100 Ω. Find the ratio of the speed of the machine as a generator to the speed as a motor, if line current is 80 A in both cases. [1.08] (Electrical Engineering-II, Bombay Univ. April. 1977, Madras Univ. Nov. 1978)

11. A 20-kW d.c. shunt generator delivering rated output at 1000 r.p.m. has a terminal voltage of 500 V. The armature resistance is 0.1 Ω, voltage drop per brush is 1 volt and the field resistance is 500 Ω. Calculate the speed at which the machine will run as a motor taking an input of 20 kW from a 500 V d.c. supply. [976.1 r.p.m.] (Elect. Engg-I Bombay Univ. 1975)

12. A 4-pole, 250-V, d.c. shunt motor has a lap-connected armature with 960 conductors. The flux per pole is \(2 \times 10^{-2}\) Wb. Calculate the torque developed by the armature and the useful torque in newton-metre when the current taken by the motor is 30 A. The armature resistance is 0.12 ohm and the field resistance is 125 Ω. The rotational losses amount to 825 W. [85.5 N-m; 75.3 N-m] (Electric Machinery-I, Madras Univ. Nov. 1979)
Example 29.29. The armature circuit resistance of a 18.65 kW 250-V series motor is 0.1 Ω, the brush voltage drop is 3V, and the series field resistance is 0.05. When the motor takes 80 A, speed is 600 r.p.m. Calculate the speed when the current is 100 A.


Solution.

\[ E_{b1} = 250 - 80 (0.1 + 0.05) - 3 = 235 \text{ V} \]

\[ E_{b2} = 250 - 100 (0.1 + 0.05) - 3 = 232 \text{ V} \]

Since \( \Phi \propto I_a \), hence, \( \Phi_1 \propto 80, \Phi_2 \propto 100, \frac{\Phi_1}{\Phi_2} = 80/100 \)

Now

\[ \frac{N_2}{N_1} = \frac{E_{b2} \times \Phi_1}{E_{b1} \times \Phi_2} \quad \text{or} \quad \frac{N_2}{600} = \frac{232 \times 80}{235 \times 100} \implies N_2 = 474 \text{ r.p.m.} \]

Example 29.30. A 220-volt d.c. series motor is running at a speed of 800 r.p.m. and draws 100 A. Calculate at what speed the motor will run when developing half the torque. Total resistance of the armature and field is 0.1 ohm. Assume that the magnetic circuit is unsaturated.


Solution.

\[ \frac{N_2}{N_1} = \frac{E_{b2} \times \Phi_1}{E_{b1} \times \Phi_2} = \frac{E_{b2} \times I_{a1}}{E_{b1} \times I_{a2}} \quad \text{ (.:. \: \Phi \propto I_a)} \]

Since field is unsaturated, \( T_a \propto \Phi I_a \propto I_a^2 \).

\[ \text{or} \quad \frac{T_2}{T_1} = \left( \frac{I_{a2}}{I_{a1}} \right)^2 \quad \text{or} \quad 1/2 = \left( \frac{I_{a2}}{I_{a1}} \right)^2 \quad \text{so} \quad I_{a1} = I_{a2}/\sqrt{2} = 70.7 \text{ A} \]

\[ E_{b1} = 220 - 100 \times 0.1 = 210 \text{ V} \; \quad E_{b2} = 220 - 0.1 \times 70.7 = 212.9 \text{ V} \]

\[ \therefore \quad \frac{N_2}{800} = \frac{212.9 \times 100}{210 \times 70.7} \implies N_2 = 1147 \text{ r.p.m.} \]

Example 29.31. A 4-pole d.c. motor runs at 600 r.p.m. on full load taking 25 A at 450 V. The armature is lap-wound with 500 conductors and flux per pole is expressed by the relation

\[ \Phi = (1.7 \times 10^{-2} \times I^{0.5}) \text{ weber} \]

where \( I \) is the motor current. If supply voltage and torque are both halved, calculate the speed at which the motor will run. Ignore stray losses.

(Elect. Machines, Nagpur Univ. 1993)

Solution. Let us first find \( R_a \)

Now

\[ N = \frac{E_b}{Z \Phi} \left( \frac{60 \text{ A}}{P} \right) \text{r.p.m.} \]

\[ 600 = \frac{E_b}{1.7 \times 10^{-2} \times 25 \times 500} \times 60 \times 4 \]

\[ \therefore \quad E_b = 10 \times 1.7 \times 10^{-2} \times 5 \times 500 = 425 \text{ V} \]

\[ I_a R_a = 450 - 425 = 25 \text{ V} \; \quad R_a = 25/25 = 1.0 \Omega \]

Now in the 1st Case

\[ T_1 \propto \Phi_1 I_1 \quad \therefore \quad T_1 \propto 1.7 \times 10^{-2} \times \sqrt{25} \times 25 \]

Similarly

\[ T_2 \propto 1.7 \times 10^{-2} \times \sqrt{1 \times 1} \; \quad \text{Now} \quad T_1 = 2T_2 \]

\[ \therefore \quad 1.7 \times 10^{-2} \times 125 = 1.7 \times 10^{-2} \times I^{3/2} \times 2 \quad \therefore \quad I = (125/2)^{2/3} = 15.75 \text{ A} \]

\[ E_{b1} = 425 \text{ V} \; \quad E_{b2} = 225 - (15.75 \times 1) = 209.3 \text{ V} \]

Using the relation

\[ \frac{N_2}{N_1} = \frac{E_{b2} \times \Phi_1}{E_{b1} \times \Phi_2} \quad \text{we have} \]
29.12. Motor Characteristics

The characteristic curves of a motor are those curves which show relationships between the following quantities.

1. **Torque and armature current** \( i.e. T_a/I_a \) characteristic. It is known as *electrical characteristic*.
2. **Speed and armature current** \( i.e. N/I_a \) characteristic.
3. **Speed and torque** \( i.e. N/T_a \) characteristic. It is also known as *mechanical characteristic*. It can be found from (1) and (2) above.

While discussing motor characteristics, the following two relations should always be kept in mind:

\[
T_a \propto \Phi I_a \quad \text{and} \quad N \propto \frac{E_b}{\Phi}
\]

29.13. Characteristics of Series Motors

1. **\( T_a/I_a \) Characteristic.** We have seen that \( T_a \propto \Phi I_a \). In this case, as field windings also carry the armature current, \( \Phi \propto I_a \) up to the point of magnetic saturation. Hence, before saturation,

\[
T_a \propto \Phi I_a \quad \text{and} \quad \therefore \quad T_a \propto I_a^2
\]

At light loads, \( I_a \) and hence \( \Phi \) is small. But as \( I_a \) increases, \( T_a \) increases as the square of the current. Hence, \( T_a/I_a \) curve is a parabola as shown in Fig. 29.14. After saturation, \( \Phi \) is almost independent of \( I_a \) hence \( T_a \propto I_a \) only. So the characteristic becomes a straight line. The shaft torque \( T_{sh} \) is less than armature torque due to stray losses. It is shown dotted in the figure. So we conclude that (prior to magnetic saturation) on heavy loads, a series motor exerts a torque proportional to the square of armature current. Hence, in cases where huge starting torque is required for accelerating heavy masses quickly as in hoists and electric trains etc., series motors are used.

![Fig. 29.14](image1)

![Fig. 29.15](image2)

![Fig. 29.16](image3)

2. **\( N/I_a \) Characteristics.** Variations of speed can be deduced from the formula:

\[
N \propto \frac{E_b}{\Phi}
\]

Change in \( E_b \), for various load currents is small and hence may be neglected for the time being. With increased \( I_a \), \( \Phi \) also increases. Hence, speed varies inversely as armature current as shown in Fig. 29.15.

When load is heavy, \( I_a \) is large. Hence, speed is low (this decreases \( E_b \) and allows more armature current to flow). But when load current and hence \( I_a \) falls to a small value, speed becomes dangerously high. Hence, a series motor should never be started without some mechanical (not belt-driven) load on it otherwise it may develop excessive speed and get damaged due to heavy centrifugal forces so produced. It should be noted that series motor is a variable speed motor.

3. **\( N/T_a \) or mechanical characteristic.** It is found from above that when speed is high, torque is low and *vice-versa*. The relation between the two is as shown in Fig. 29.16.

1. $T_a/I_a$ Characteristic

Assuming $\Phi$ to be practically constant (though at heavy loads, $\phi$ decreases somewhat due to increased armature reaction) we find that $T_a \propto I_a$.

Hence, the electrical characteristic as shown in Fig. 29.17, is practically a straight line through the origin. Shaft torque is shown dotted. Since a heavy starting load will need a heavy starting current, shunt motor should never be started on (heavy) load.

2. $N/I_a$ Characteristic

If $\Phi$ is assumed constant, then $N \propto E_b$. As $E_b$ is also practically constant, speed is, for most purposes, constant (Fig. 29.18).

![Graphs showing $T_a$, $T_{sh}$, and $N$ versus $I_a$](image)

But strictly speaking, both $E_b$ and $\Phi$ decrease with increasing load. However, $E_b$ decreases slightly more than $\phi$ so that on the whole, there is some decrease in speed. The drop varies from 5 to 15% of full-load speed, being dependent on saturation, armature reaction and brush position. Hence, the actual speed curve is slightly drooping as shown by the dotted line in Fig. 29.18. But, for all practical purposes, shunt motor is taken as a constant-speed motor.

Because there is no appreciable change in the speed of a shunt motor from no-load to full-load, it may be connected to loads which are totally and suddenly thrown off without any fear of excessive speed resulting. Due to the constancy of their speed, shunt motors are suitable for driving shafting, machine tools, lathes, wood-working machines and for all other purposes where an approximately constant speed is required.

3. $N/T_a$ Characteristic can be deduced from (1) and (2) above and is shown in Fig. 29.19.

29.15. Compound Motors

These motors have both series and shunt windings. If series excitation helps the shunt excitation i.e. series flux is in the same direction (Fig. 29.20); then the motor is said to be cummulatively compounded. If on the other hand, series field opposes the shunt field, then the motor is said to be differentially compounded.

The characteristics of such motors lie in between those of shunt and series motors as shown in Fig. 29.21.

(a) Cumulative-compound Motors

Such machines are used where series characteristics are required and where, in addition,
the load is likely to be removed totally such as in some types of coal cutting machines or for driving heavy machine tools which have to take sudden cuts quite often. Due to shunt windings, speed will not become excessively high but due to series windings, it will be able to take heavy loads. In conjunction with fly-wheel (functioning as load equalizer), it is employed where there are sudden temporary loads as in rolling mills. The fly-wheel supplies its stored kinetic energy when motor slows down due to sudden heavy load. And when due to the removal of load motor speeds up, it gathers up its kinetic energy.

Compound-wound motors have greatest application with loads that require high starting torques or pulsating loads (because such motors smooth out the energy demand required of a pulsating load). They are used to drive electric shovels, metal-stamping machines, reciprocating pumps, hoists and compressors etc.

(b) Differential-compound Motors
Since series field opposes the shunt field, the flux is decreased as load is applied to the motor. This results in the motor speed remaining almost constant or even increasing with increase in load (because, \( N \propto E_b/(\Phi) \)). Due to this reason, there is a decrease in the rate at which the motor torque increases with load. Such motors are not in common use. But because they can be designed to give an accurately constant speed under all conditions, they find limited application for experimental and research work.

One of the biggest drawbacks of such a motor is that due to weakening of flux with increases in load, there is a tendency towards speed instability and motor running away unless designed properly.

**Example 29.32.** The following results were obtained from a static torque test on a series motor:

<table>
<thead>
<tr>
<th>Current (A)</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque (N*m)</td>
<td>128.8</td>
<td>230.5</td>
<td>349.8</td>
<td>46.2</td>
</tr>
</tbody>
</table>

Deduce the speed/torque curve for the machine when supplied at a constant voltage of 460 V. Resistance of armature and field winding is 0.5 \( \Omega \). Ignore iron and friction losses.
Solution. Taking the case when input current is 20 A, we have

Motor input \( = 460 \times 20 = 9,200 \) W
Field and armature Cu loss
\( = 20^2 \times 0.5 = 200 \) W

Ignoring iron and friction losses,
output \( = 9,200 - 200 = 9,000 \) W

Now, \( T_{sh} \times 2\pi N = \text{Output in watts.} \)
\( \therefore \quad 128.8 \times 2\pi \times N = 9,000 \)
\( \therefore \quad N = \frac{9,000}{2\pi \times 128.8} \)
\( = 11.12 \text{ r.p.s.} = 667 \text{ r.p.m.} \)

Similar calculations for other values of current are tabulated below:

<table>
<thead>
<tr>
<th>Current (A)</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input (W)</td>
<td>9,200</td>
<td>13,800</td>
<td>18,400</td>
<td>23,000</td>
</tr>
<tr>
<td>( I^2R ) loss (W)</td>
<td>200</td>
<td>450</td>
<td>800</td>
<td>1,250</td>
</tr>
<tr>
<td>Output (W)</td>
<td>9,200</td>
<td>13,350</td>
<td>17,600</td>
<td>21,850</td>
</tr>
<tr>
<td>Speed (r.p.m.)</td>
<td>667</td>
<td>551</td>
<td>480</td>
<td>445</td>
</tr>
<tr>
<td>Torque (N-m)</td>
<td>128.8</td>
<td>230.5</td>
<td>349.8</td>
<td>469.2</td>
</tr>
</tbody>
</table>

From these values, the speed/torque curve can be drawn as shown in Fig. 29.22.

Example 29.33. A fan which requires 8 h.p. (5.968 kW) at 700 r.p.m. is coupled directly to a d.c. series motor. Calculate the input to the motor when the supply voltage is 500 V, assuming that power required for fan varies as the cube of the speed. For the purpose of obtaining the magnetisation characteristics, the motor was running as a self-excited generator at 600 r.p.m. and the relationship between the terminal voltage and the load current was found to be as follows:

| Load current (A) | 7 | 10.5 | 14 | 27.5 |
| Terminal voltage (V) | 347 | 393 | 434 | 458 |

The resistance of both the armature and field windings of the motor is 3.5 \( \Omega \) and the core, friction and other losses may be assumed to be constant at 450 W for the speeds corresponding to the above range of currents at normal voltage.

Solution. Let us, by way of illustration, calculate the speed and output when motor is running off a 500-V supply and taking a current of 7 A.

Series voltage drop \( = 7 \times 3.5 = 24.5 \) V
Generated or back e.m.f. \( E_b = 500 - 24.5 = 475.5 \) V

The motor speed is proportional to \( E_b \) for a given current. For a speed of 600 r.p.m. and a current of 7 A, the generated e.m.f. is 347 V. Hence,
\( N = 600 \times \frac{475.5}{347} = 823 \text{ r.p.m.} \)

Power to armature \( = E_bJ_a = 475.5 \times 7 = 3,329 \) W
Output = Armature power - 450 = 3,329 - 450 = 2,879 W = 2.879 kW

Power required by the fan at 823 r.p.m. is \( = 5.968 \times \frac{823^2}{700^2} = 9.498 \) kW

These calculations are repeated for the other values of current in the following table.

| Input current (A) | 7 | 10.5 | 14 | 27.5 |
| Series drop (V) | 24.5 | 36.7 | 49 | 96.4 |
| Back e.m.f. (V) | 475.5 | 463.3 | 451 | 403.6 |
E.M.F. at 600 r.p.m. (V) 347 393 434 458
Speed N (r.p.m.) 823 707 623 528
Armature power (W) 3329 4870 6310 11,100
Motor output (kW) 2.879 4.420 5.860 10.65
Power required by fan (kW) 9.698 6.146 4.222 2.566

In Fig. 29.23 (i) the motor output in kW and (ii) power required by fan in kW against input current is plotted. Since motor output equals the input to fan, hence the intersection point of these curves gives the value of motor input current under the given conditions.

\[ \therefore \text{Motor input} = 500 \times 12 = 6000 \text{ W} \]

29.16. Performance Curves

(a) Shunt Motor

In Fig. 29.24 the four essential characteristics of a shunt motor are shown i.e. torque, current, speed and efficiency, each plotted as a function of motor output power. These are known as the performance curves of a motor.

It is seen that shunt motor has a definite no-load speed. Hence, it does not ‘run away’ when load is suddenly thrown off provided the field circuit remains closed. The drop in speed from no-load to full-load is small, hence this motor is usually referred to as constant speed motor. The speed for any load within the operating range of the motor can be readily obtained by varying the field current by means of a field rheostat.

The efficiency curve is usually of the same shape for all electric motors and generators. The shape of efficiency curve and the point of maximum efficiency can be varied considerably by the designer, though it is advantageous to have an efficiency curve which is fairly flat, so that there is little change in efficiency between load and 25% overload and to have the maximum efficiency as near to the full load as possible.

It will be seen from the curves, that a certain value of current is required even when output is zero. The motor input under no-load conditions goes to meet the various losses occurring within the machine.
As compared to other motors, a shunt motor is said to have a lower starting torque. But this should not be taken of mean that a shunt motor is incapable of starting a heavy load. Actually, it means that series and compound motors are capable of starting heavy loads with less excess of current inputs over normal values than the shunt motors and that consequently the depreciation on the motor will be relatively less. For example, if twice full load torque is required at start, then shunt motor draws twice the full-load current \( T_a \propto I_a \) or \( I_a \propto \sqrt{T_a} \) whereas series motor draws only approximately one and a half times the full load current \( T_a \propto I_a^2 \) or \( I_a \propto \sqrt{T_a} \).

The shunt motor is widely used with loads that require essentially constant speed but where high starting torques are not needed. Such loads include centrifugal pumps, fans, winding reels conveyors and machine tools etc.

(b) Series Motor

The typical performance curves for a series motor are shown in Fig. 29.25.

It will be seen that drop in speed with increased load is much more prominent in series motor than in a shunt motor. Hence, a series motor is not suitable for applications requiring a substantially constant speed.

For a given current input, the starting torque developed by a series motor is greater than that developed by a shunt motor. Hence, series motors are used where huge starting torques are necessary i.e. for street cars, cranes, hoists and for electric-railway operation. In addition to the huge starting torque, there is another unique characteristic of series motors which makes them especially desirable for traction work i.e. when a load comes on a series motor, it responds by decreasing its speed (and hence, \( E_b \)) and supplies the increased torque with a small increase in current. On the other hand a shunt motor under the same conditions would hold its speed nearly constant and would supply the required increased torque with a large increase of input current. Suppose that instead of a series motor, a shunt motor is used to drive a street car. When the car ascend a grade, the shunt motor maintains the speed for the car at approximately the same value it had on the level ground, but the motor tends to take an excessive current. A series motor, however, automatically slows down on such a grade because of increased current demand, and so it develops more torque at reduced speed. The drop in speed permits the motor to develop a large torque with but a moderate increase of power. Hence, under the same load conditions, rating of the series motor would be less than for a shunt motor.
### 29.17. Comparison of Shunt and Series Motors

**a) Shunt Motors**

The different characteristics have been discussed in Art. 29.14. It is clear that

1. the speed of a shunt motor is sufficiently constant.
2. for the same current input, its starting torque is not as high as that of series motor. Hence, it is used.
3. When the speed has to be maintained approximately constant from N.L. to F.L. *i.e.* for driving a line of shafting etc.
4. When it is required to drive the load at various speeds, any one speed being kept constant for a relatively long period *i.e.* for individual driving of such machines as lathes. The shunt regulator enables the required speed control to be obtained easily and economically.

#### Summary of Applications

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**b) Series Motors**

The operating characteristics have been discussed in Art 29.13. These motors

1. have a relatively huge starting torques.
2. have good accelerating torque.
3. have low speed at high loads and dangerously high speed at low loads.

Hence, such motors are used

1. when a large starting torque is required *i.e.* for driving hoists, cranes, trams etc.
2. when the motor can be directly coupled to a load such as a fan whose torque increases with speed.

3. if constancy of speed is not essential, then, in fact, the decrease of speed with increase of load has the advantage that the power absorbed by the motor does not increase as rapidly as the torque. For instance, when torque is doubled, the power approximately increases by about 50 to 60% only. (\( \therefore I_a \propto \sqrt{T_a} \)).

4. a series motor should not be used where there is a possibility of the load decreasing to a very small value. Thus, it should not be used for driving centrifugal pumps or for a belt-drive of any kind.

### 29.18. Losses and Efficiency

The losses taking place in the motor are the same as in generators. These are (i) Copper losses (ii) Magnetic losses and (iii) Mechanical losses.

The condition for maximum power developed by the motor is

\[ I_a R_a = V/2 = E_{fr} \]

The condition for maximum efficiency is that armature Cu losses are equal to constant losses. (Art. 26.39).

### 29.19. Power Stages

The various stages of energy transformation in a motor and also the various losses occurring in it are shown in the flow diagram of Fig. 29.26.

Overall or commercial efficiency \( \eta_c = \frac{C}{A} \), Electrical efficiency \( \eta_e = \frac{B}{A} \), Mechanical efficiency \( \eta_m = \frac{C}{B} \).

The efficiency curve for a motor is similar in shape to that for a generator (Art. 24.35).

![Fig. 29.26](image)

It is seen that \( A - B = \) copper losses and \( B - C = \) iron and friction losses.

**Example 29.34.** One of the two similar 500-V shunt machines A and B running light takes 3 A. When A is mechanically coupled to B, the input to A is 3.5 A with B unexcited and 4.5 A when B is separately-excited to generate 500 V. Calculate the friction and windage loss and core loss of each machine.

(From **Electric Machinery-I, Madras Univ. 1985**)

**Solution.** When running light, machine input is used to meet the following losses (i) armature Cu loss (ii) shunt Cu loss (iii) iron loss and (iv) mechanical losses i.e. friction and windage losses. Obviously, these no-load losses for each machine equal 500 x 3 = 1500 W.
(a) With B unexcited
In this case, only mechanical losses take place in B, there being neither Cu loss nor iron-loss because B is unexcited. Since machine A draws 0.5 A more current.
Friction and windage loss of \( B = 500 \times 0.5 = 250 \) W

(b) With B excited
In this case, both iron losses as well as mechanical losses take place in machine B. Now, machine A draws, 4.5 – 3 = 1.5 A more current.
Iron and mechanical losses of \( B = 1.5 \times 500 = 750 \) W
Iron losses of \( B = 750 - 250 = 500 \) W

Example 29.35. A 220 V shunt motor has an armature resistance of 0.2 ohm and field resistance of 110 ohm. The motor draws 5 A at 1500 r.p.m. at no load. Calculate the speed and shaft torque if the motor draws 52 A at rated voltage. (Elect. Machines Nagpur Univ. 1993)

Solution.
\[
I_{sh} = \frac{220}{110} = 2 \text{ A}; I_{a1} = 5 - 2 = 3 \text{ A}; I_{a2} = 52 - 2 = 50 \text{ A}
\]
\[
E_{b1} = 220 - 3 \times 0.2 = 219.4 \text{ V}; E_{b2} = 220 - 50 \times 0.2 = 210 \text{ V}
\]
\[
\frac{N_2}{1500} = \frac{210}{219.4}; N_2 = 1436 \text{ r.p.m.} \quad (\because \Phi_1 = \Phi_2)
\]

For finding the shaft torque, we will find the motor output when it draws a current of 52 A. First we will use the no-load data for finding the constant losses of the motor.
No load motor input = \( 220 \times 5 = 1100 \) W; Arm. Cu loss = \( 3^2 \times 0.2 = 2 \) W
\[\therefore\] Constant or standing losses of the motor = \( 1100 - 2 = 1098 \) W
When loaded, arm. Cu loss = \( 50^2 \times 0.2 = 500 \) W
Hence, total motor losses = \( 1098 + 500 = 1598 \) W
Motor input on load = \( 220 \times 52 = 11,440 \) W; output = \( 11,440 - 1598 = 9842 \) W
\[\therefore\] \( T_{sh} = 9.55 \times \text{output}/N = 9.55 \times 9842/1436 = 65.5 \text{ N-m} \)

Example 29.36. 250 V shunt motor on no load runs at 1000 r.p.m. and takes 5 amperes. Armature and shunt field resistances are 0.2 and 250 ohms respectively. Calculate the speed when loaded taking a current of 50 A. The armature reaction weakens the field by 3%. (Elect. Engg.-I Nagpur Univ. 1993)

Solution.
\[
I_{sh} = \frac{250}{250} = 1 \text{ A}; I_{a1} = 5 - 1 = 4 \text{ A}; I_{a2} = 50 - 1 = 49 \text{ A}
\]
\[
E_{b1} = 250 - 4 \times 0.2 = 249.2 \text{ V}; E_{b2} = 250 - 49 \times 0.2 = 240.2 \text{ V}
\]
\[
\frac{N_2}{1000} = \frac{240.2}{249.2 \times \Phi_1}; N_2 = 944 \text{ r.p.m.}
\]

Example 29.37. A 500 V d.c. shunt motor takes a current of 5 A on no-load. The resistances of the armature and field circuit are 0.22 ohm and 250 ohm respectively. Find (a) the efficiency when loaded and taking a current of 100 A (b) the percentage change of speed. State precisely the assumptions made. (Elect. Engg.-I, M.S. Univ. Baroda 1987)

Solution. No-Load condition
\[
I_{sh} = \frac{500}{250} = 2 \text{ A}; I_{a0} = 5 - 2 = 3 \text{ A}; E_{b0} = 500 - (3 \times 0.22) = 499.34 \text{ V}
\]
Arm. Cu loss = \( 3^2 \times 0.22 = 2 \) W; Motor input = \( 500 \times 5 = 2500 \) W
Constant losses = \( 2500 - 2 = 2498 \) W
It is assumed that these losses remain constant under all load conditions.

Load condition
(a) Motor current = 100 A; \( I_a = 100 - 2 = 98 \) A; \( E_b = 500 - (98 \times 0.22) = 478.44 \) V
Arm. Cu loss = \(98^2 \times 0.22 = 2110\) W, Total losses = 2110 + 2498 = 4608 W
Motor input = \(500 \times 100 = 50,000\) W, Motor output = 50,000 – 4,608 = 45,392 W
Motor \(\eta = \frac{45,392}{50,000} = 0.908\) or 90.8% \(\frac{N}{N_0} = \frac{E_b}{E_{b0}} = \frac{478.44}{499.34} = \frac{N - N_0}{N_0} = \frac{-20.9}{499.34} = -0.0418\) or -4.18%

**Example 29.38.** A 250 V d.c. shunt motor runs at 1000 r.p.m. while taking a current of 25 A. Calculate the speed when the load current is 50 A if armature reaction weakens the field by 3%. Determine torques in both cases.

\[R_a = 0.2\ \text{ohm};\ R_f = 250\ \text{ohms}\]

Voltage drop per brush is 1 V.  
(Elect. Machines Nagpur Univ. 1993)

**Solution.**

\[I_{sh} = \frac{250}{250} = 1 \text{ A};\ I_{al} = 25 - 1 = 24 \text{ A}\]

\[E_{bh} = 250 - \text{arm. drop} - \text{brush drop}\]

\[= 250 - 24 \times 0.2 - 2 = 243.2 \text{ V}\]

\[I_{a2} = 50 - 1 = 49 \text{ A};\ E_{b2} = 250 - 49 \times 0.2 - 2 = 238.2 \text{ V}\]

\[\frac{N_2}{1000} = \frac{238.2}{243.2} \times \frac{\Phi_1}{0.97}\ \text{r.p.m.};\ N_2 = 1010 \text{ r.p.m.}\]

\[T_{a1} = 9.55 \times \frac{E_{b1}}{I_{a1}} = 9.55 \times \frac{243.2}{24}/1000 = 55.7 \text{ N-m}\]

\[T_{a2} = 9.55 \times \frac{E_{b2}}{I_{a2}} = 9.55 \times \frac{238.2}{49}/1010 = 110.4 \text{ r.p.m.}\]

**Example 29.39.** A d.c. shunt machine while running as generator develops a voltage of 250 V at 1000 r.p.m. on no-load. It has armature resistance of 0.5 Ω and field resistance of 250 Ω. When the machine runs as motor, input to it at no-load is 4 A at 250 V. Calculate the speed and efficiency of the machine when it runs as a motor taking 40 A at 250 V. Armature reaction weakens the field by 4%.

(Electrical Technology, Aligarh Muslim Univ. 1989)

**Solution.**

\[\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}\]

Now, when running as a generator, the machine gives 250 V at 1000 r.p.m. If this machine was running as motor at 1000 r.p.m., it will, obviously, have a back e.m.f. of 250 V produced in its armature. Hence \(N_1 = 1000\) r.p.m. and \(E_{b1} = 250\) V.

When it runs as a motor, drawing 40 A, the back e.m.f. induced in its armature is

\[E_{b2} = 250 - (40 - 1) \times 0.5 = 230.5 \text{ V};\ \text{Also } \Phi_2 = 0.96 \Phi_1, N_2 = ?\]

Using the above equation we have

\[\frac{N_2}{1000} = \frac{230.5}{250} \times \frac{\Phi_1}{0.96} ; N_2 = 960 \text{ r.p.m.}\]

**Efficiency**

No-load input represents motor losses which consists of

(a) armature Cu loss = \(I_a^2 R_a\) which is variable.

(b) constant losses \(W_c\) which consists of (i) shunt Cu loss (ii) magnetic losses and (iii) mechanical losses.

No-load input or total losses = \(250 \times 4 = 1000\) W

Arm. Cu loss = \(I_a^2 R_a = 3^2 \times 0.5 = 4.5\) W, \(\therefore\) \(W_c = 1000 - 4.5 = 995.5\) W

When motor draws a line current of 40 A, its armature current is \((40 - 1) = 39\) A

Arm. Cu loss = \(39^2 \times 0.5 = 760.5\) W; Total losses = 760.5 + 955.5 = 1756 W

Input = \(250 \times 40 = 10,000\) W; output = \(10,000 - 1756 = 8,244\) W

\(\therefore\)

\(\eta = \frac{8,244 \times 100}{10,000} = 82.44\%\)
Example 29.40. The armature winding of a 4-pole, 250 V d.c. shunt motor is lap connected. There are 120 slots, each slot containing 8 conductors. The flux per pole is 20 mWb and current taken by the motor is 25 A. The resistance of armature and field circuit are 0.1 and 125 Ω respectively. If the rotational losses amount to be 810 W find,

(i) gross torque (ii) useful torque and (iii) efficiency. (Elect. Machines Nagpur Univ. 1993)

Solution. \( I_{sh} = 250/125 = 2 \) A; \( I_a = 25 - 2 = 23 \) A; \( E_b = 250 - (23 \times 0.1) = 247.7 \) V

Now, \( E_b = \frac{\Phi ZN}{60} \left( \frac{P}{A} \right) \) \[ 247.7 = \frac{20 \times 10^{-3} \times 960 \times N}{60} \left( \frac{4}{4} \right); N = 773 \text{ r.p.m.} \]

(i) Gross torque or armature torque \( T_a = 9.55 \frac{E_b I_a}{N} = 9.55 \times \frac{247.7 \times 23}{773} = 70.4 \) N·m

(ii) Arm Cu loss = \( 23^2 \times 0.1 = 53 \) W; Shunt Cu loss = \( 250 \times 2 = 500 \) W

Rotational losses = 810 W; Total motor losses = 810 + 500 + 53 = 1363 W

Motor input = \( 250 \times 25 = 6250 \) W; Motor output = \( 6250 - 1363 = 4887 \) W

\( T_{sh} = 9.55 \times \text{output}/N = 9.55 \times \frac{4887}{773} = 60.4 \) N-m

(iii) Efficiency = \( 4887/6250 = 0.782 = 78.2\% \)

Example 29.41. A 20-hp (14.92 kW); 230-V, 1150-r.p.m, 4-pole, d.c. shunt motor has a total of 620 conductors arranged in two parallel paths and yielding an armature circuit resistance of 0.2 Ω. When it delivers rated power at rated speed, it draws a line current of 74.8 A and a field current of 3 A. Calculate (i) the flux per pole (ii) the torque developed (iii) the rotational losses (iv) total losses expressed as a percentage of power. (Electrical Machinery-I, Banglore Univ. 1987)

Solution.

\( I_a = 74.8 - 3 = 71.8 \) A; \( E_b = 230 - 71.8 \times 0.2 = 215.64 \) V

(i) Now, \( E_b = \frac{\Phi ZN}{60} \left( \frac{P}{A} \right) \) \[ 215.64 = \frac{\Phi \times 620 \times 1150}{60} \left( \frac{4}{2} \right); \Phi = 9 \text{ mWb} \]

(ii) Armature Torque, \( T_a = 9.55 \times 215.64 \times 71.8/1150 = 128.8 \) N·m

(iii) Driving power in armature = \( E_b I_a = 215.64 \times 71.8 = 15,483 \) W

Output = 14,920 W; Rotational losses = 15,483 - 14,920 = 563 W

(iv) Motor input = \( VI = 230 \times 74.8 = 17,204 \) W; Total loss = 17,204 - 14,920 = 2,284 W

Losses expressed as percentage of power input = 2284/17,204 = 0.133 or 13.3%

Example 29.42. A 7.46 kW, 250-V shunt motor takes a line current of 5 A when running light. Calculate the efficiency as a motor when delivering full load output, if the armature and field resistance are 0.5 Ω and 250 Ω respectively. At what output power will the efficiency be maximum? Is it possible to obtain this output from the machine? (Electrotechnics-II, M.S. Univ. Baroda 1985)

Solution. When loaded lightly

Total motor input (or total no-load losses) = 250 × 5 = 1,250 W

\( I_{sh} = 250/250 = I_a \) \[ I_a = 5 - 1 = 4 \] A

Field Cu loss = \( 250 \times 1 = 250 \) W; Armature Cu loss = \( 4^2 \times 0.5 = 8 \) W

\( \therefore \) Iron losses and friction losses = \( 1250 - 250 - 8 = 992 \) W

These losses would be assumed constant.

Let \( I_a \) be the full-load armature current, then armature input is = \( (250 \times I_a) \) W

F.L. output = \( 7.46 \times 1000 = 7,460 \) W

The losses in the armature are:

(i) Iron and friction losses = 992 W

(ii) Armature Cu loss = \( I_a^2 \times 0.05 \) W \[ 250 I_a = 7,460 + 992 + I_a^2 \times 0.5 \]
or \( 0.5 I_a^2 - 250 I_a + 8,452 = 0 \) \quad \therefore \quad I_a = 36.5 \ A

\[ \therefore \quad \text{F.L. input current} = 36.5 + 1 = 37.5 \ A \quad \text{; Motor input} = 250 \times 37.5 \ W \]

\[ \therefore \quad \text{F.L. output} = 7,460 \ W \]

\[ \therefore \quad \text{F.L. efficiency} = 7460 \times 100/250 \times 37.5 = 79.6\% \]

Now, efficiency is maximum when armature Cu loss equals constant loss.

\[ i.e. \quad I_a^2 R_d = I_a^2 \times 0.5 = (1,250 - 8) = 1,242 \ W \text{ or } I_a = 49.84 \ A \]

\[ \therefore \quad \text{Armature input} = 250 \times 49.84 = 12,460 \ W \]

Armature Cu loss = 49.84\(^2\) \times 0.5 = 1242 \ W; \text{ Iron and friction losses} = 992 \ W

\[ \therefore \quad \text{Armature output} = 12,460 - (1,242 + 992) = 10,226 \ W \]

\[ \therefore \quad \text{Output power} = 10,226 \ W = 10.226 \ kW \]

As the input current for maximum efficiency is beyond the full-load motor current, it is never realised in practice.

**Example 29.43.** A d.c. series motor drives a load, the torque of which varies as the square of the speed. Assuming the magnetic circuit to be remain unsaturated and the motor resistance to be negligible, estimate the percentage reduction in the motor terminal voltage which will reduce the motor speed to half the value it has on full voltage. What is then the percentage fall in the motor current and efficiency? Stray losses of the motor may be ignored.

(Electrical Engineering-III, Pune Univ. 1987)

**Solution.** \( T_s \propto \Phi I_a \propto I_a^2 \). Also, \( T_s \propto N^2 \). Hence \( N^2 \propto I_a^2 \) or \( N \propto I_a \)

\[ \therefore \quad N_1 \propto I_{a1} \text{ and } N_2 \propto I_{a2} \text{ or } N_2/N_1 = I_{a2}/I_{a1} \]

Since,

\[ N_2/N_1 = 1/2 \quad \therefore \quad I_{a2}/I_{a1} = 1/2 \text{ or } I_{a2} = I_{a1}/2 \]

Let \( V_1 \) and \( V_2 \) be the voltages across the motor in the two cases. Since motor resistance is negligible, \( E_{b1} = V_1 \) and \( E_{b2} = V_2 \). Also \( \Phi_1 \propto I_{a1} \) and \( \Phi_2 \propto I_{a2} \) or \( \Phi_1/\Phi_2 = I_{a1}/I_{a2} = I_{a1} \times 2/I_{a1} = 2 \)

Now,

\[ \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \text{ or } \frac{1}{2} = \frac{V_2}{V_1} \times 2 \text{ or } \frac{V_2}{V_1} = \frac{1}{4} \]

\[ \therefore \quad \frac{V_1 - V_2}{V_1} = \frac{4 - 1}{4} = 0.75 \]

\[ \therefore \quad \text{Percentage reduction in voltage} = \frac{V_1 - V_2}{V_1} \times 100 = 0.75 \times 100 = 75\% \]

Percentage change in motor current = \( \frac{I_{a1} - I_{a2}}{I_{a1}} \times 100 = \frac{I_{a1} - I_{a1}/2}{I_{a1}} \times 100 = 50\% \)

**Example 29.44.** A 6-pole, 500-V wave-connected shunt motor has 1200 armature conductors and useful flux/pole of 20 mWb. The armature and field resistance are 0.5 \( \Omega \) and 250 \( \Omega \) respectively. What will be the speed and torque developed by the motor when it draws 20 A from the supply mains? Neglect armature reaction. If magnetic and mechanical losses amount to 900 W, find (i) useful torque (ii) output in kW and (iii) efficiency at this load.

**Solution.** (i)

\[ I_{sh} = 500/250 = 2 \ A \quad \therefore \quad I_a = 20 - 2 = 18 \ A \]

\[ \therefore \quad E_b = 500 - (18 \times 0.5) = 491 \ V; \text{ Now, } E_b = \frac{\Phi ZN}{60} \times \left( \frac{P}{A} \right) \text{ volt} \]

\[ \therefore \quad 491 = \frac{20 \times 10^{-3} \times 1200 \times N}{60} \times \left( \frac{6}{2} \right) ; \quad N = 410 \ \text{r.p.m. (approx.)} \]

Now

\[ T_a = 9.55 \frac{E_b I_a}{N} = 9.55 \frac{491 \times 18}{410} = 206 \text{ N} \cdot \text{m} \]
Armature Cu loss = \(18^2 \times 0.5 = 162\) W; Field Cu loss = \(500 \times 2 = 1000\) W
Iron and friction loss = \(900\) W; Total loss = \(162 + 1000 + 900 = 2,062\) W
Motor input = \(500 \times 20 = 10,000\) W

\[\begin{align*}
(i) & \quad T_{sh} = 9.55 \times \frac{7938}{410} = 184.8\ N\cdot m \\
(ii) & \quad Output = 10,000 - 2062 = 7,938\ kW \\
(iii) & \quad \% \eta = \frac{Output}{Input} \times 100 = \frac{7,938 \times 100}{10,000} = 0.794 = 79.4\%
\end{align*}\]

Example 29.45. A 50-h.p. (37.3 kW), 460-V d.c. shunt motor running light takes a current of 4 A and runs at a speed of 660 r.p.m. The resistance of the armature circuit (including brushes) is 0.3 \(\Omega\) and that of the shunt field circuit 270 \(\Omega\).

Determine when the motor is running at full load
(i) the current input (ii) the speed. Determine the armature current at which efficiency is maximum. Ignore the effect of armature reaction. (Elect. Technology Punjab, Univ. 1991)

Solution. \(I_{sh} = 460/270 = 1.7\) A; Field Cu loss = \(460 \times 1.7 = 783\) W

When running light
\(I_a = 4 - 1.7 = 2.3\) A; Armature Cu loss = \(2.3^2 \times 0.3 = 1.5\) W (negligible)
No-load armature input = \(460 \times 2.3 = 1,058\) W

As armature Cu loss is negligible, hence 1,058 W represents iron, friction and windage losses which will be assumed to be constant.

Let full-load armature input current be \(I_a\). Then
Armature input = \(460 I_a\) W; Armature Cu loss = \(I_a^2 \times 0.3\) W
Output = \(37.3\) kW = \(37,300\) W
\[\begin{align*}
\therefore & \quad 460 I_a = 37,300 + 1,058 + 0.3 I_a^2 \text{ or } 0.3 I_a^2 - 460 I_a + 38,358 = 0 \\
\therefore & \quad I_a = 88.5\ A \\
(i) & \quad \text{Current input} = 88.5 + 1.7 = 90.2\ A \\
(ii) & \quad E_{bi} = 460 - (2.3 \times 0.3) = 459.3\ V; E_{b2} = 460 - (88.5 \times 0.3) = 433.5\ V \\
\therefore & \quad N_2 = 660 \times 433.5/459.3 = 624\ r.p.m.
\end{align*}\]

For maximum efficiency, \(I_a^2 R_a = \) constant losses (Art. 24.37)
\[\begin{align*}
\therefore & \quad I_a^2 \times 0.3 = 1058 + 783 = 1,841 \therefore I_a = (1841/0.3)^{1/2} = 78.33\ A
\end{align*}\]

Tutorial Problems 29.3

1. A 4-pole 250-V, d.c. series motor has a wave-wound armature with 496 conductors. Calculate
(a) the gross torque (b) the speed
(b) the output torque and (d) the efficiency, if the motor current is 50 A

The value of flux per pole under these conditions is 22 mWb and the corresponding iron, friction and windage losses total 810 W. Armature resistance = 0.19 \(\Omega\), field resistance = 0.14 \(\Omega\).

\[\begin{align*}
[(a) & \quad 173.5\ N\cdot m (b) \quad 642\ r.p.m. (c) \quad 161.4\ N\cdot m (d) \quad 86.9\%]
\end{align*}\]

2. On no-load, a shunt motor takes 5 A at 250 V, the resistances of the field and armature circuits are
250 \(\Omega\) and 0.1 \(\Omega\) respectively. Calculate the output power and efficiency of the motor when the total supply current is 81 A at the same supply voltage. State any assumptions made.

[18.5 kW; 91%. It is assumed that windage, friction and eddy current losses are independent of the current and speed]
3. A 230 V series motor is taking 50 A. Resistance of armature and series field windings is 0.2 Ω and 0.1 Ω respectively. Calculate:
   (a) brush voltage  
   (b) back e.m.f.  
   (c) power wasted in armature  
   (d) mechanical power developed
   
   \[
   \begin{align*}
   (a) & \ 215 \text{ V} \\
   (b) & \ 205 \text{ V} \\
   (c) & \ 500 \text{ W} \\
   (d) & \ 13.74 \text{ h.p.} \\
   \end{align*}
   \]  
   (10.25 kW)

4. Calculate the shaft power of a series motor having the following data; overall efficiency 83.5%, speed 550 r.p.m. when taking 65 A; motor resistance 0.2 Ω, flux per pole 25 mWb, armature winding lap with 1200 conductor.
   (15.66 kW)

5. A shunt motor running on no-load takes 5 A at 200 V. The resistance of the field circuit is 150 Ω and of the armature 0.1 Ω. Determine the output and efficiency of motor when the input current is 120 A at 200 V. State any conditions assumed.  
   (89.8%)  

6. A d.c. shunt motor with interpoles has the following particulars:
   Output power; 8,952 kW, 440-V, armature resistance 1.1 Ω, brush contact drop 2 V, interpole winding resistance 0.4 Ω, shunt resistance 650 Ω, resistance in the shunt regulator 50 Ω. Iron and friction losses on full-load 450 W. Calculate the efficiency when taking the full rated current of 24 A.  
   (85%)

7. A d.c. series motor on full-load takes 50 A from 230 V d.c. mains. The total resistance of the motor is 0.22 Ω. If the iron and friction losses together amount to 5% of the input, calculate the power delivered by the motor shaft. Total voltage drop due to the brush contact is 2 A.  
   (10.275 kW)

8. A 2-pole d.c shunt motor operating from a 200 V supply takes a full-load current of 35 A, the no-load current being 2 A. The field resistance is 500 Ω and the armature has a resistance of 0.6 Ω. Calculate the efficiency of the motor on full-load. Take the brush drop as being equal to 1.5 V per brush arm. Neglect temperature rise.  
   [Rajiv Gandhi Tech. Univ. Bhopal, 2000] (82.63%)

**OBJECTIVE TESTS – 29**

1. In a d.c. motor, undirectional torque is produced with the help of
   (a) brushes  
   (b) commutator  
   (c) end-plates  
   (d) both (a) and (b)

2. The counter e.m.f. of a d.c. motor
   (a) often exceeds the supply voltage  
   (b) aids the applied voltage  
   (c) helps in energy conversion  
   (d) regulates its armature voltage

3. The normal value of the armature resistance of a d.c. motor is
   (a) 0.005  
   (b) 0.5  
   (c) 10  
   (d) 100
   (Grad. I.E.T.E. June 1987)

4. The \( E_f/V \) ratio of a d.c. motor is an indication of its
   (a) efficiency  
   (b) speed regulation  
   (c) starting torque  
   (d) Running Torque
   (Grad. I.E.T.E. June 1987)

5. The mechanical power developed by the armature of a d.c. motor is equal to
   (a) armature current multiplied by back e.m.f.  
   (b) power input minus losses  
   (c) power output multiplied by efficiency  
   (d) power output plus iron losses

6. The induced e.m.f. in the armature conductors of a d.c. motor is
   (a) sinusoidal  
   (b) trapezoidal  
   (c) rectangular  
   (d) alternating

7. A d.c. motor can be looked upon as d.c. generator with the power flow
   (a) reduced  
   (b) reversed  
   (c) increased  
   (d) modified

8. In a d.c. motor, the mechanical output power actually comes from
   (a) field system  
   (b) air-gap flux  
   (c) back e.m.f.  
   (d) electrical input power

9. The maximum torque of d.c. motors is limited by
   (a) commutation  
   (b) heating
10. Which of the following quantity maintains the same direction whether a d.c. machine runs as a generator or as a motor?
(a) induced e.m.f.  (b) armature current
(c) field current    (d) supply current

11. Under constant load conditions, the speed of a d.c. motor is affected by
(a) field flux        (b) armature current
(c) back e.m.f.       (d) both (b) and (c)

12. It is possible to increase the field flux and, at the same time, increase the speed of a d.c. motor if a ......... is held constant.
(a) applied voltage
(b) torque
(c) Armature circuit resistance
(d) armature current

13. The current drawn by a 120-V d.c. motor of armature resistance 0.5 Ω and back e.m.f. 110 V is ......... ampere.
(a) 20  (b) 240
(c) 220  (d) 5

14. The shaft torque of a d.c. motor is less than its armature torque because of ......... losses.
(a) copper   (b) mechanical
(c) iron     (d) rotational

15. A d.c. motor develops a torque of 200 N-m at 25 rps. At 20 rps it will develop a torque of ......... N-m.
(a) 200  (b) 160
(c) 250  (d) 128

16. Neglecting saturation, if current taken by a series motor is increased from 10 A to 12 A, the percentage increase in its torque is ......... percent
(a) 20  (b) 44
(c) 30.5  (d) 16.6

17. If load on a d.c. shunt motor is increased, its speed is decreased due primarily to
(a) increase in its flux
(b) decrease in back e.m.f.
(c) increase in armature current
(d) increase in brush drop

18. If the load current and flux of a d.c. motor are held constant and voltage applied across its armature is increased by 10 per cent, its speed will
(a) decrease by about 10 per cent
(b) remain unchanged

19. If the pole flux of a d.c. motor approaches zero, its speed will
(a) approach zero
(b) approach infinity
(c) no change due to corresponding change in back e.m.f.
(d) approach a stable value somewhere between zero and infinity.

20. If the field circuit of a loaded shunt motor is suddenly opened
(a) it would race to almost infinite speed
(b) it would draw abnormally high armature current
(c) circuit breaker or fuse will open the circuit before too much damage is done to the motor
(d) torque developed by the motor would be reduced to zero.

21. Which of the following d.c. motor would be suitable for drives requiring high starting torque but only fairly constant speed such as crushers?
(a) shunt   (b) series
(c) compound   (d) permanent magnet

22. A d.c. shunt motor is found suitable to drive fans because they require
(a) small torque at start up
(b) large torque at high speeds
(c) practically constant voltage
(d) both (a) and (b)

23. Which of the following load would be best driven by a d.c. compound motor?
(a) reciprocating pump
(b) centrifugal pump
(c) electric locomotive
(d) fan

24. As the load is increased, the speed of a d.c. shunt motor
(a) increases proportionately
(b) remains constant
(c) increases slightly
(d) reduces slightly

25. Between no-load and full-load, ......... motor develops the least torque
(a) series
(b) shunt
(c) cumulative compound
(d) differential compound
26. The \( T/T_a \) graph of a d.c. series motor is a
   (a) parabola from no-load to overload
   (b) straight line throughout
   (c) parabola throughout
   (d) parabola up to full-load and a straight line at overloads.

27. As compared to shunt and compound motors, series motor has the highest torque because of its comparatively .......... at the start.
   (a) lower armature resistance
   (b) stronger series field
   (c) fewer series turns
   (d) larger armature current

28. Unlike a shunt motor, it is difficult for a series motor to stall under heavy loading because
   (a) it develops high overload torque
   (b) its flux remains constant
   (c) it slows down considerably
   (d) its back e.m.f. is reduced to almost zero.

29. When load is removed, ........... motor will run at the highest speed.
   (a) shunt
   (b) cumulative-compound
   (c) differential compound
   (d) series

30. A series motor is best suited for driving
   (a) lathes
   (b) cranes and hoists
   (c) shears and punches
   (d) machine tools

31. A 220 V shunt motor develops a torque of 54 N-m at armature current of 10 A. The torque produced when the armature current is 20 A, is
   (a) 54 N-m
   (b) 81 N-m
   (c) 108 N-m
   (d) None of the above

32. The d.c. series motor should never be switched on at no load because
   (a) the field current is zero
   (b) The machine does not pick up
   (c) The speed becomes dangerously high
   (d) It will take too long to accelerate.
   (Grad. I.E.T.E. June 1988)

33. A shunt d.c. motor works on a.c. mains
   (a) unsatisfactorily
   (b) satisfactorily
   (c) not at all
   (d) none of the above

34. A 200 V, 10 A motor could be rewound for 100 V, 20 A by using .......... as many turns per coil of wire, having .......... the cross-sectional area.
   (a) twice, half
   (b) thrice, one third
   (c) half, twice
   (d) four times, one-fourth

Answers

1. (d) 2. (c) 3. (b) 4. (a) 5. (a) 6. (a) 7. (b) 8. (d) 9. (a) 10. (a) 11. (a)
12. (d) 13. (a) 14. (d) 15. (a) 16. (b) 17. (b) 18. (c) 19. (b) 20. (c) 21. (c) 22. (d)
23. (a) 24. (d) 25. (a) 26. (d) 27. (b) 28. (a) 29. (d) 30. (b) 31. (c) 32. (c) 33. (a)
Learning Objectives

- Factors Controlling Motor Speed
- Speed Control of Shunt Motors
- Speed Control of Series Motors
- Merits and Demerits of Rheostatic Control Method
- Series-Parallel Control
- Electric Braking
- Electric Braking of Shunt Motor
- Electric Braking of Series Motors
- Electronic Speed control Method for D.C. Motors
- Uncontrolled Rectifiers
- Controlled Rectifiers
- Thyristor Choppers
- Thyristor Inverters
- Thyristor Speed Control of Separately-excited D.C. Motor
- Thyristor Speed Control of D.C. Series Motor
- Full-wave Speed Control of a Shunt Motor
- Thyristor Control of a Shunt Motor
- Thyristor Speed Control of a Series D.C. Motor
- Necessity of a Starter
- Shunt Motor Starter
- Three-point Starter
- Four-point Starter
- Starting and Speed Control of Series Motors
- Grading of Starting Resistance
- Shunt Motors
- Series Motor Starters
- Thyristor Controller Starters

DC motor speed controller control the speed of any common dc motor rated upto 100 V. It operates on 5V to 15 V.
30.1. Factors Controlling Motor Speed

It has been shown earlier that the speed of a motor is given by the relation

\[ N = \frac{V - I_a R_a}{Z \Phi} \cdot \frac{(A)}{(P)} = K \frac{V - I_a R_a}{\Phi} \text{ r.p.s.} \]

where \( R_a \) = armature circuit resistance.

It is obvious that the speed can be controlled by varying (i) flux/pole, \( \Phi \) (Flux Control) (ii) resistance \( R_a \) of armature circuit (Rheostatic Control) and (iii) applied voltage \( V \) (Voltage Control). These methods as applied to shunt, compound and series motors will be discussed below.

30.2. Speed Control of Shunt Motors

(i) Variation of Flux or Flux Control Method

It is seen from above that \( N \propto 1/\Phi \). By decreasing the flux, the speed can be increased and vice versa. Hence, the name flux or field control method. The flux of a d.c. motor can be changed by changing \( I_{sh} \) with help of a shunt field rheostat (Fig. 30.1). Since \( I_{sh} \) is relatively small, shunt field rheostat has to carry only a small current, which means \( F R \) loss is small, so that rheostat is small in size. This method is, therefore, very efficient. In non-interpo lar machine, the speed can be increased by this method in the ratio 2:1. Any further weakening of flux \( \Phi \) adversely affects the communication and hence puts a limit to the maximum speed obtainable with the method. In machines fitted with interpoles, a ratio of maximum to minimum speed of 6:1 is fairly common.

Example 30.1. A 500 V shunt motor runs at its normal speed of 250 r.p.m. when the armature current is 200 A. The resistance of armature is 0.12 ohm. Calculate the speed when a resistance is inserted in the field reducing the shunt field to 80% of normal value and the armature current is 100 ampere.


Solution. \( E_{b1} = 500 - 200 \times 0.12 = 476 \text{ V} \); \( E_{b2} = 500 - 100 \times 0.12 = 488 \text{ V} \)

\[ \Phi_2 = 0.8 \Phi_1; N_1 = 250 \text{ rpm}; \quad \frac{N_2}{N_1} = ? \]

Now,

\[ \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \quad \text{or} \quad \frac{N_2}{N_1} = \frac{488}{476} \times \frac{0.8 \Phi_1}{\Phi_2} \]

\[ N_2 = 320 \text{ r.p.m.} \]

Example 30.2. A 250 volt d.c. shunt motor has armature resistance of 0.25 ohm, on load it takes an armature current of 50 A and runs at 750 r.p.m. If the flux of motor is reduced by 10% without changing the load torque, find the new speed of the motor.

(Elect. Eng.-II, Pune Univ. 1987)

Solution.

\[ \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \]

Now, \( T_a \propto \Phi I_a \). Hence \( T_{a1} \propto \Phi_1 I_{a1} \) and \( T_{a2} \propto \Phi_2 I_{a2} \).

Since \( T_{a1} = T_{a2} \quad \therefore \quad \Phi_1 I_{a1} = \Phi_2 I_{a2} \)

\[ \Phi_2 = 0.9 \Phi_1 \quad \therefore \quad 50 \Phi_1 = 0.9 \Phi_1, I_{a2} = 55.6 \text{ A} \]

\[ \therefore \quad E_{b1} = 250 - (50 \times 0.25) = 237.5 \text{ V} \text{; } E_{b2} = 250 - (55.6 \times 0.25) = 231.1 \text{ V} \]

\[ \therefore \quad \frac{N_2}{750} = \frac{231.1}{237.5} \times \frac{0.9 \Phi_1}{\Phi_1} \quad ; \quad N_2 = 811 \text{ r.p.m.} \]

Example 30.3. Describe briefly the method of speed control available for dc motors.

A 230 V d.c. shunt motor runs at 800 r.p.m. and takes armature current of 50 A. Find resistance
to be added to the field circuit to increase speed to 1000 r.p.m. at an armature current of 80 A. Assume flux proportional to field current. Armature resistance = 0.15 Ω and field winding resistance = 250 Ω.

(Elect. Technology, Hyderabad Univ. 1991)

**Solution.**

\[
\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \times \frac{l_{sh2}}{l_{sh1}} \quad \text{since flux \(\propto\) field current}
\]

\[
E_{b1} = 230 - (50 \times 0.15) = 222.5 \text{ V; } E_{b2} = 230 - (80 \times 0.15) = 218 \text{ V}
\]

Let

\[
R_l = \text{total shunt resistance} = (250 + R) \text{ where } R \text{ is the additional resistance}
\]

\[
i_{sh1} = 230/250 = 0.92 \text{ A, } i_{sh2} = 230/R_l \quad ; \quad N_1 = 800 \text{ r.p.m.}; \quad N = 1000 \text{ r.p.m.}
\]

\[
\therefore \quad \frac{1000}{800} = \frac{218}{222.5} \times \frac{0.92}{230/R_l} \quad ; \quad R_l = 319 \text{ \Omega} \quad \therefore \quad R = 319 - 250 = 69 \text{ \Omega}
\]

\[
i_{sh2} = \frac{230}{319} = 0.721
\]

Ratio of torque in two cases = \[
\frac{T_2}{T_1} = \frac{i_{sh2}}{i_{sh1}} \frac{l_{a2}}{l_{a1}} = \frac{0.721 \times 80}{0.92 \times 50} = 1.254
\]

Example 30.4. A 250 V, d.c. shunt motor has shunt field resistance of 250 Ω and an armature resistance of 0.25 Ω. For a given load torque and no additional resistance included in the shunt field circuit, the motor runs at 1500 r.p.m. drawing an armature current of 20 A. If a resistance of 250 Ω is inserted in series with the field, the load torque remaining the same, find out the new speed and armature current. Assume the magnetisation curve to be linear.

(Electrical Engineering-1, Bombay Univ. 1987)

**Solution.** In this case, the motor speed is changed by changing the flux.

Now,

\[
\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}
\]

Since it is given that magnetisation curve is linear, it means that flux is directly proportional to shunt current. Hence \[
\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{l_{sh1}}{l_{sh2}} \quad \text{where } E_{b2} = V - I_{a2} R_a \quad \text{and } E_{b1} = V - I_{a1} R_a
\]

Since load torque remains the same  \[
\therefore \quad T_o \propto \Phi_1 I_{a1} \propto \Phi_2 I_{a2} \quad \text{or } \quad \Phi_1 I_{a1} = \Phi_2 I_{a2}
\]

\[
\therefore \quad I_{a2} = I_{a1} \times \frac{\Phi_1}{\Phi_2} = I_{a1} \times \frac{l_{sh1}}{l_{sh2}}
\]

Now,

\[
i_{sh1} = 250/250 = 1 \text{ A;} \quad i_{sh2} = 250/(250 + 250) = 1/2 \text{ A}
\]

\[
I_{a2} = 20 \times \frac{1}{1/2} = 40 \text{ A} \quad \therefore \quad E_{b2} = 250 - (40 \times 0.25) = 240 \text{ V and}
\]

\[
E_{b1} = 250 - (20 \times 0.25) = 245 \text{ V} \quad \therefore \quad \frac{N_2}{1500} = \frac{240}{245} \times \frac{1}{1/2}
\]

\[
N_2 = 2,930 \text{ r.p.m.}
\]

Example 30.5. A 250 V, d.c. shunt motor has an armature resistance of 0.5 Ω and a field resistance of 250 Ω. When driving a load of constant torque at 600 r.p.m., the armature current is 20 A. If it is desired to raise the speed from 600 to 800 r.p.m., what resistance should be inserted in the shunt field circuit? Assume that the magnetic circuit is unsaturated.

(Elect. Engg. AMIETE, June 1992)

**Solution.**

\[
\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}
\]

Since the magnetic circuit is unsaturated, it means that flux is directly proportional to the shunt current.
\[ N = \frac{E_b}{E_{b1}} \times \frac{I_{sh1}}{I_{sh2}} \text{ where } E_{b1} = V - I_{a1} R_a \text{ and } E_{b2} = V - I_{a2} R_a \]

Since motor is driving at load of constant torque,

\[ T_a \propto \Phi_1 I_{a1} \propto \Phi_2 I_{a2} \quad \therefore \quad \Phi_2 I_{a2} = \Phi_1 I_{a1} \quad \text{or} \quad I_{a2} = I_{a1} \times \frac{\Phi_1}{\Phi_2} = I_{a1} \times \frac{I_{sh1}}{I_{sh2}} \]

Now,

\[ I_{sh1} = \frac{250}{250} = 1 \, \text{A} \quad \text{and} \quad I_{sh2} = \frac{250}{R_t} \]

where \( R_t \) is the total resistance of the shunt field circuit.

\[ I_{a2} = 20 \times \frac{1}{250/R_t} = \frac{2R_t}{25} \quad \therefore \quad E_{b2} = 250 - \left( \frac{2R_t}{25} \times 0.5 \right) = 250 - \left( \frac{R_t}{25} \times 0.5 \right) \cdot \frac{800}{600} = \frac{250 - (R_t/25)}{240} \times \frac{1}{250/R_t} \]

\[ 0.04R_t^2 - 250R_t + 80,000 = 0 \]

or

\[ R_t = \frac{250 \pm \sqrt{62,500 - 12,800}}{0.08} = \frac{27}{0.08} = 337.5 \, \Omega \]

Additional resistance required in the shunt field circuit \( = 337.5 - 250 = 87.5 \, \Omega \).

**Example 30.6.** A 220 V shunt motor has an armature resistance of 0.5 \( \Omega \) and takes a current of 40 A on full-load. By how much must the main flux be reduced to raise the speed by 50% if the developed torque is constant? (Elect. Machines, AMIE, Sec B, 1991)

**Solution.** Formula used is \( \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \). Since torque remains constant,

\[ \Phi_1 I_{a1} = \Phi_2 I_{a2} \quad \therefore \quad I_{a2} = I_{a1} \cdot \frac{\Phi_1}{\Phi_2} = 40 \times x \text{ where } x = \frac{\Phi_1}{\Phi_2} \]

\[ E_{b1} = 220 - (40 \times 0.5) = 200 \, \text{V} \quad \text{and} \quad E_{b2} = 220 - (40 \times 0.5 \times 0.5) = (220 - 20x) \, \text{V} \]

\[ \frac{N_2}{N_1} = \frac{3}{2} \quad \text{given} \quad \therefore \quad \frac{3}{2} = \frac{(220 - 20x)}{200} \times x \quad \therefore \quad x^2 - 11x + 15 = 0 \]

or

\[ x = \frac{11 \pm \sqrt{121 - 60}}{2} = \frac{11 \pm 7.81}{2} = 9.4^* \quad \text{or} \quad 1.6 \]

\[ \therefore \quad \frac{\Phi_1}{\Phi_2} = 1.6 \quad \text{or} \quad \frac{\Phi_2}{\Phi_1} = 1 \]

\[ \therefore \quad \frac{\Phi_1 - \Phi_2}{\Phi_1} = \frac{1.6 - 1}{1.6} = \frac{3}{8} \quad \therefore \quad \text{percentage change in flux} = \frac{3}{8} \times 100 = 37.5\% \]

**Example 30.7.** A 220-V, 10-kW, 2500 r.p.m. shunt motor draws 41 A when operating at rated conditions. The resistances of the armature, compensating winding, interpole winding and shunt field winding are respectively 0.2 \( \Omega \), 0.05 \( \Omega \), 0.1 \( \Omega \) and 110 \( \Omega \). Calculate the steady-state values of armature current and motor speed if pole flux is reduced by 25%, a 1 \( \Omega \) resistance is placed in series with the armature and the load torque is reduced by 50%.

**Solution.** \( I_{sh} = \frac{220}{110} = 2 \, A \); \( I_{a1} = 41 - 2 = 39 \, A \) (Fig. 30.2)

\[ T_1 \propto \Phi_1 I_{a1} \text{ and } T_2 \propto \Phi_2 I_{a2} \]

\[ \therefore \quad \frac{T_2}{T_1} = \frac{\Phi_1 \times I_{a1}}{\Phi_2 \times I_{a2}} \]

* This figure is rejected as it does not give the necessary increase in speed.
or \( \frac{1}{2} = \frac{3}{4} \times \frac{I_{a2}}{39} \) \( \therefore \) \( I_{a2} = 26 \text{ A} \)

\[ E_{b1} = 220 - 39 (0.2 + 0.1 + 0.05) = 206.35 \text{ V} \]
\[ E_{b2} = 220 - 26 (1 + 0.35) = 184.9 \text{ V} \]

Now, \( \frac{N_2}{2500} = \frac{184.9}{206.35} \times \frac{4}{3} ; N_2 = 2987 \text{ r.p.m.} \)

**Example 30.8.** A 220 V, 15 kW, 850 r.p.m. shunt motor draws 72.2 A when operating at rated condition. The resistances of the armature and shunt field are 0.25 \( \Omega \) and 100 \( \Omega \) respectively. Determine the percentage reduction in field flux in order to obtain a speed of 1650 r.p.m. when armature current drawn is 40 A.

**Solution.**
\[ I_{sh} = \frac{220}{100} = 2.2 \text{ A} ; I_{a1} = 72.2 - 2.2 = 70 \text{ A} \]
\[ E_{b1} = 220 - 70 \times 0.25 = 202.5 \text{ V} , \ E_{b2} = 220 - 40 \times 0.25 = 210 \text{ V} . \]

Now, \[ \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \]
\[ \therefore \quad \Phi_2 = 0.534 \Phi_1 ; \]
\[ \therefore \quad \text{Reduction in field flux} = \frac{\Phi_1 - 0.534 \Phi_1}{\Phi_1} \times 100 = 46.6\% . \]

**Example 30.9.** A 220 V shunt motor has an armature resistance of 0.5 ohm and takes an armature current of 40 A on a certain load. By how much must the main flux be reduced to raise the speed by 50% if the developed torque is constant? Neglect saturation and armature reaction.

(=Elect. Machines, AMIE, Sec B, 1991=)

**Solution.**
\[ T_1 \propto \Phi_1 \times I_{a1} \rightarrow \Phi_1 I_{a1} , \quad \text{and} \quad T_2 \propto \Phi_2 I_{a2} \quad \text{Since,} \quad T_1 = T_2 \]
\[ \therefore \Phi_2 I_{a2} = \Phi_1 I_{a1} \quad \text{or} \quad \Phi_2 = I_{a2} I_{a1} \]
\[ E_{b1} = 220 - 40 \times 0.5 = 200 \text{ V} ; \quad E_{b2} = 220 - 0.5 I_{a2} \]

Now, \[ \frac{E_{b2}}{E_{b1}} = \frac{N_2 \Phi_2}{N_1 \Phi_1} \quad \text{or} \quad \frac{220 - 0.5 I_{a2}}{200} = \frac{1.5 N_1 	imes 40}{N_1 I_{a2}} \]
\[ \therefore \quad I_{a2} - 40 I_{a2} + 24,000 = 0 \quad \text{or} \quad I_{a2} = 63.8 \text{ A} \]
\[ \therefore \quad \frac{\Phi_2}{\Phi_1} = \frac{40}{63.8} = 0.627 \quad \text{or} \quad \Phi_2 = 0.627 \Phi_1 = 62.7\% \text{ of} \Phi_1 \]

**Example 30.10.** A d.c. shunt motor takes an armature current of 20 A from a 220 V supply. Armature circuit resistance is 0.5 ohm. For reducing the speed by 50%, calculate the resistance required in the series, with the armature, if

(a) the load torque is constant
(b) the load torque is proportional to the square of the speed. (Sambalpur Univ., 1998)

**Solution.**
\[ E_{b1} = V - I_a r_a = 220 - 20 \times 0.5 = 210 \text{ V} \]
\[ 210 \propto N_1 \]

(a) **Constant Load torque**
In a shunt motor, flux remains constant unless there is a change in terminal voltage or there is a change in the field-circuit resistance. If torque is constant, armature-current then must remain constant. \( I_a = 20 \text{ amp} \)
With an external armature-circuit resistor of \( R \) ohms, \( 20 \times (R + 0.5) = 220 - E_{b2} \)
The speed required now is 0.5 \( N_1 \).
With constant flux, \( E_b \propto \text{speed}. \) Hence, \( 210 \propto N_1 \)
\[
E_{b2} \propto 0.5 \times N_1, \quad E_{b2} = 105
\]
\[
R + 0.5 = \frac{220 - 105}{20} = 5.75, \quad \text{giving} \quad R = 5.25 \text{ ohms}
\]

**(b) Load torque is proportional to the square of speed.**

With constant flux, Developed Torque at \( N_1 \) r.p.m. \( \propto \) \( I_a \)
\[
T_{m1} \propto 20
\]

From the Load Side, \( T_{L1} \propto N_1^2 \)
Since
\[
T_{m1} = T_{L1}
\]
\[
20 \propto N_1^2
\]

At 50% speed, Load Torque, \( T_{L2} \propto (0.5 N_1)^2 \)

For motor torque,
\[
T_{m2} \propto I_{a2}
\]

Since
\[
T_{L2} = T_{m2}, \quad I_{a2} \propto (0.5 N_1)^2
\]

From eqn. (a) and (b) above
\[
\frac{I_{a2}}{I_{a1}} = 0.25, \quad I_{a2} = 5 \text{ amp}
\]
\[
\frac{E_{b2}}{E_{bl}} = \frac{220 - I_{a2} (R + 0.5)}{210} = 0.5 N_1
\]
\[
220 - I_{a2} (R + 0.5) = 0.5 \times 210
\]
\[
R + 0.5 = \frac{220 - 105}{5} = 23, \quad R = 22.5 \text{ ohms}
\]

**Check:** With the concept of armature power output: (applied here for part (b) only as an illustration).

Armature power–output = \( E_b \times I_a = T \times \omega \)

When \( T \propto (\text{speed})^2 \), \( E_b I_a = k_1 T_\omega = k_2 \omega^3 \)

At \( N_1 \) r.p.m. \( 210 \times 20 = K_3 N_1^3 \)

...**(c)**

With constant flux, at half speed
\[
E_{b2} = 105
\]
\[
105 \times I_{a2} = K_3 (0.5 N_1)^3
\]

From eqs. (c) and (d),
\[
\frac{105 \times I_{a2}}{210 \times 20} = \frac{0.125 \times N_1^3}{N_1^3}, \quad \text{giving} \quad I_{a2} = 5 \text{ amp}
\]

This gives \( R = 22.5 \text{ ohms} \).

**Example 30.11.** A 250 V shunt motor runs at 1000 r.p.m. at no-load and takes 8 A. The total armature and shunt field resistances are respectively 0.2 ohm, 250 ohm. Calculate the speed when loaded and taking 50 A. Assume the flux to be constant.

(Nagpur Univ. Summer 2000)

**Solution.** The current distribution is shown in Fig. 30.3.

At no load, \( I_L = 8 \text{ amp}, \quad I_f = 1 \text{ amp}, \)
Hence, \( I_a = 7 \text{ amp} \)
\[
E_{b0} = 250 - 7 \times 0.2 = 248.6 \text{ volts},
\]
\[
= K \phi \times 1000
\]
\[
\therefore \quad K \phi = 0.2486
\]
At load,
\[ I_a = 49 \text{ amp} \]
\[ E_{bl} = 250 - 49 \times 0.2 = 240.2 \]
\[ N_1 = \frac{240.2}{0.2486} = 966.2 \text{ r.p.m.} \]

Notes: (i) The assumption of constant flux has simplified the issue. Generally, armature reaction tends to weaken the flux and then the speed tends to increase slightly.
(ii) The no load armature current of 7 amp is required to overcome the mechanical losses of motor as well as driven load, at about 1000 r.p.m.

**Example 30.12.** A 240 V d.c. shunt motor has an armature-resistance of 0.25 ohm, and runs at 1000 r.p.m., taking an armature current 40 A. It is desired to reduce the speed to 800 r.p.m.

(i) If the armature current remains the same, find the additional resistance to be connected in series with the armature-circuit.

(ii) If, with the above additional resistance in the circuit, armature current decreases to 20 A, find the speed of the motor.

*Bhartiar Univ., November 1997*

**Solution.**
\[ E_b = 240 - 0.25 \times 40 = 230 \text{ V} \]
\[ E_{b2} \approx 800 \]

From (a) and (b),
\[ E_{b2} = \frac{800}{1000} \times 230 = 184 \]
\[ 240 - (R + 0.25) \times 40 = 184 \]
\[ R = 1.15 \text{ ohm} \]

(ii)
\[ E_{b3} = 240 - 20 (1.40) = 212 \]
\[ E_{b3} = \frac{N_3}{1000} \times 230 = 212 \]
\[ N_3 = \frac{212}{230} \times 1000 = 922 \text{ r.p.m.} \]

**Example 30.13.** A 7.48 kW, 220 V, 990 r.p.m. shunt motor has a full load efficiency of 88%, the armature resistance is 0.08 ohm and shunt field current is 2 A. If the speed of this motor is reduced to 450 r.p.m. by inserting a resistance in the armature circuit, find the motor output, the armature current, external resistance to be inserted in the armature circuit and overall efficiency. Assume the load torque to remain constant.

*Nagpur Univ., November 1998*

**Solution.** With an output of 7.48 kW and an efficiency of 88%, the input power is 8.50 kW. Losses are 1.02 kW.

\[ \text{Input Current} = \frac{85000}{220} = 38.64 \text{ A} \]
\[ \text{Armature-Current} = 38.64 - 2.00 = 36.64 \text{ A} \]
\[ \text{Power Loss in the shunt field circuit} = 220 \times 2 = 440 \text{ W} \]
\[ \text{Copper-Loss in armature-circuit} = 36.64^2 \times 0.08 = 107.4 \text{ W} \]
\[ \text{No-Load-Loss at 900 r.p.m.} = 1020 - 107.4 - 440 = 473 \text{ W} \]
\[ \text{At 900 r.p.m. Back-emf} = E_{bl} = 200 - (36.64 \times 0.08) \approx 217.3 \text{ V} \]
\[ \text{Motor will run at 450 r.p.m. with flux per pole kept constant, provided the back-emf} = E_{b2} \]
\[ = (459/900) \times 217.1 \text{ V} = 108.5 \text{ V} \]

There are two simplifying assumptions in this case, which must be stated before further calculations:

1. Load-torque is constant,
2. No Load Losses are constant.

(These statements can be different which leads to variations in the next steps of calculations.)

For constant load-torque, the condition of constant flux per pole results into constant armature current, which is 36.64 A.

With an armature current of 36.64 A, let the external resistance required for this purpose be \( R \).
\[ 36.64 \times R = 217.1 - 108.5 = 108.6 \text{ V}, R = 2.964 \text{ ohms} \]
Total Losses
Total $i^2r$ loss in armature = $36.64^2 \times (2.964 + 0.08) = 4086$ W
Field-copper-loss + No load loss = $440 + 473 = 913$
Total Loss = 4999 W
Total Output = $8500 - 4999 = 3501$ W
Hence, Efficiency = $(3501/8500) \times 100 = 41.2\%$

(Note: Because of missing data and clarification while making the statements in the question, there can be variations in the assumption and hence in the final solutions.)

Example 30.14. A d.c. shunt motor supplied at 230 V runs at 990 r.p.m. Calculate the resistance required in series with the armature circuit to reduce the speed to 500 r.p.m. assuming that armature current is 25 amp.

(Nagpur Univ., November 1997)

Solution. It is assumed that armature resistance is to

(a) At 900 r.p.m. : $E_{b1} = 230 = K \times 900$
(b) At 500 r.p.m. : $E_{b2} = K \times 500$

Therefore, 
$E_{b2} = E_{b1} \times 500/900 = 127.8$ volts

The difference between $E_{b1}$ and $E_{b2}$ must be the drop in the external resistance to be added to the armature circuit for the purpose of reducing the speed to 500 r.p.m.

$E_{b1} - E_{b2} = 25 \times R$

Example 30.15. A 220 V d.c. shunt motor has an armature resistance of 0.4 ohm and a field circuit resistance of 200 ohms. When the motor is driving a constant-torque load, the armature-current is 20 A, the speed being 600 r.p.m. It is desired to run the motor at 900 r.p.m. by inserting a resistance in the field circuit. Find its value, assuming that the magnetic circuit is not saturated.

(Nagpur Univ., November 1996)

Solution. (i) At 600 r.p.m. $i_1 = 220/200 = 1.1$ amp
$E_{b1} = 220 - (20 \times 0.4) = 212$ volts
$T_L = K_1 \times 1.1 \times 20$

The back e.m.f. = $212 = K_2 \times 1.1 \times 600$

or $K_2 = 212/660 = 0.3212$

(ii) At 900 r.p.m. : $T_L = K_1 \times i_{f2} \times I_{a2}$

Due to constant load torque,

$i_{f2} \times I_{a2} = 1.1 \times 20 = 22$

$E_{b2} = 220 - (0.4 I_{a2}) = K_2 \times i_{f2} \times 900 = 289 i_{f2}$

$220 - (0.4 \times 22/i_{f2}) = 289 i_{f2}$

Guess for approximate value of $i_{f2}$: Neglecting armature -resistance drop and saturation, 50% rise in speed is obtained with proportional decrease in $i_{f2}$ related by

$\frac{600}{900} \approx \frac{i_{f2}}{i_{f1}}$ giving $i_{f2} \approx 0.73$ amp

[In place of $i_{f2}$, $I_{a2}$ can be evaluated first. Its guess-work will give $I_{a2} \approx 1.5 \times 20 \approx 30$ amp]

Continuing with the solution of the equation to evaluate a value of $i_{f2}$, accepting that value which is near 0.73 amp, we have $i_{f2} = 0.71865$. [Note that other value of $i_{f2}$, which is 0.04235, is not acceptable.]. Corresponding $I_{a2} = 30.6$ amp. Previous shunt field current, $i_{f1} = 1.1$, $R_{f1} = 200$ Ω.

New shunt field current, $i_{f2} = 0.71865$, $R_{f2} = 220/0.71865 = 306$ Ω. Final answer is that a resistor of 106 ohms is to be added to the field circuit to run the motor at 900 r.p.m. at constant torque.
Example 30.16. A 220 V d.c. shunt motor has an armature resistance of 0.40 ohm and field-resistance of 200 ohms. It takes an armature current of 22 A and runs at 600 r.p.m. It drives a load whose torque is constant. Suggest a suitable method to raise the speed to 900 r.p.m. Calculate the value of the controllable parameter.

(Nagpur Univ., April 1998)

Solution. At 600 r.p.m. \( I_{a1} = 22 \text{ amp} \), \( N_1 = 600 \text{ r.p.m.} \), \( i_{f1} = 220/200 = 1.1 \text{ amp} \).

\[
E_{b1} = 220 - (22 \times 0.40) = 211.2 \text{ volts.}
\]

Let the Load-torque be denoted by \( T_L \), \( k_1 \) and \( k_2 \) in the equations below represent machine constants appearing in the usual emf-equation and torque-equation for the d.c. shunt motor.

\[
E_{b1} = 211.2 = k_1 \times i_{f1} \times N_1 = k_1 \times 1.10 \times 600 \quad \text{or} \quad k_1 = 211.2/660 = 0.32
\]

\[
T_L = k_2 \times i_{f1} \times I_{a1} = k_2 \times 1.10 \times 22.
\]

Since the load torque will remain constant at 900 r.p.m. also, the corresponding field current (= \( i_{f2} \)) and armature current (= \( I_{a2} \)) must satisfy the following relationships :

\[
T_L = k_2 i_{f2} I_{a2} = k_2 \times 1.10 \times 22
\]

or

\[
i_{f2} I_{a2} = 24.2
\]

And

\[
E_{b2} = 220 - (I_{a2} \times 0.40) = k_1 \times i_{f2} \times 900
\]

\[
220 - (I_{a2} \times 0.40) = 0.32 \times (24.2 \times I_{a2}) \times 900
\]

(Alternatively, the above equation can also lead to a quadratic in \( i_{f2} \).)

This leads to a quadratic equation in \( I_{a2} \):

Guess for \( i_{f2} \): Approximately, speed of a d.c. shunt motor is inversely proportional to the field current. Comparing the two speeds of 600 and 900 r.p.m., the value of \( i_{f2} \) should be approximately given by

\[
i_{f2} \equiv i_{f1} \times (600/900) = 0.733 \text{ amp}
\]

Guess for \( I_{a2} \): For approximate conclusions, armature-resistance drop can be ignored. With constant load-torque, armature-power must be proportional to the speed.

\[
\text{Armature-power at 900 r.p.m.} = \frac{900}{600} = 1.5
\]

\[
E_{b2} I_{a2} = 1.5 \times E_{b1} I_{a1}
\]

Neglecting armature-resistance drops, \( E_{b1} = V \) and \( E_{b2} = V \).

This gives

\[
I_{a2} = 1.5 \times 22 = 33 \text{ amps}
\]

Thus, out of the two roots for \( I_{a2} \), that which is close to 0.733 is acceptable. If quadratic equation for \( I_{a2} \) is being handled, that root which is near 33 amp is acceptable.

Continuing with the solution to quadratic equation for \( I_{a2} \), we have

\[
220 - 0.40 I_{a2} = 0.32 \times (24.2 \times I_{a2}) \times 900
\]

\[
220 - 0.40 I_{a2} = 6969/I_{a2}
\]

\[
I_{a2}^2 - 550 I_{a2} + 17425 = 0
\]

This gives \( I_{a2} \) as \text{either} 33.75 amp or 791.25 amp.

From the reasoning given above, acceptable root corresponds to \( I_{a2} = 33.75 \text{ amp} \).

Corresponding field current, \( i_{f2} = 24.2/33.75 = 0.717 \text{ amp} \)

Previous field circuit resistance = 200 ohms

New field circuit resistance = 220/0.717 = 307 ohms

Hence, additional resistance of 107 ohms must be added to the shunt field circuit to run the motor at 900 r.p.m. under the stated condition of constant Load torque.

Additional Check: Exact calculations for proportions of armature-power in two cases will give the necessary check.

\[
E_{b2} = 220 - (33.75 \times 0.40) = 206.5
\]
As mentioned above, while guessing the value of $I_{a2}$, the proportion of armature-power should be 1.5.

$$\frac{E_{b2} I_{a2}}{E_{b1} I_{a1}} = \frac{206.56 \times 33.75}{211.2 \times 22} = 1.50$$

Thus, the results obtained are confirmed.

**Example 30.17.** A 250 V, 25 kW d.c. shunt motor has an efficiency of 85% when running at 1000 r.p.m. on full load. The armature resistance is 0.1 ohm and field resistance is 125 ohms. Find the starting resistance required to limit the starting current to 150% of the rated current. **(Amravati Univ., 1999)**

**Solution.**

Output power = 25 kW, at full-load.

Input power = \(\frac{25,000}{0.85} = 29412\) watts

At Full load, Input Current = \(\frac{29412}{250} = 117.65\) amp

Field Current = \(\frac{250}{125} = 2\) amp

F.L. Armature Current = \(117.65 - 2 = 115.65\) amp

Limit of starting current = \(1.50 \times 115.65 = 173.5\) amp

Total resistance in armature circuit at starting

\[\frac{250}{173.5} = 1.441\] ohms

External resistance to be added to armature circuit

\[1.441 - 0.1 = 1.341\] ohm.

---

**Tutorial Problems 30.1**

1. A d.c. shunt motor runs at 900 r.p.m. from a 460 V supply when taking an armature current of 25 A. Calculate the speed at which it will run from a 230-V supply when taking an armature current of 15 A. The resistance of the armature circuit is 0.8 Ω. Assume the flux per pole at 230 V to have decreased to 75% of its value at 460 V. **[595 r.p.m.]**

2. A 250 V shunt motor has an armature resistance of 0.5 Ω and runs at 1200 r.p.m. when the armature current is 80 A. If the torque remains unchanged, find the speed and armature current when the field is strengthened by 25%. **[998 r.p.m.; 64 A]**

3. When on normal full-load, a 500 V, d.c. shunt motor runs at 800 r.p.m. and takes an armature current 42 A. The flux per pole is reduced to 75% of its normal value by suitably increasing the field circuit resistance. Calculate the speed of the motor if the total torque exerted on the armature is (a) unchanged (b) reduced by 20%.

The armature resistance is 0.6 Ω and the total voltage loss at the brushes is 2 V. **[(a) 1,042 r.p.m. (b) 1,061 r.p.m.]**

4. The following data apply to d.c. shunt motor.

Supply voltage = 460 V; armature current = 28 A; speed = 1000 r.p.m.; armature resistance = 0.72 Ω. Calculate (i) the armature current and (ii) the speed when the flux per pole is increased to 120% of the initial value, given that the total torque developed by the armature is unchanged. **[(i) 23.33 A (ii) 840 r.p.m.]**

5. A 100-V shunt motor, with a field resistance of 50 Ω and armature resistance of 0.5 Ω runs at a speed of 1,000 r.p.m. and takes a current of 10 A from the supply. If the total resistance of the field circuit is reduced to three quarters of its original value, find the new speed and the current taken from the supply. Assume that flux is directly proportional to field current. **[1,089 r.p.m.; 8.33 A]**
6. A 250 V d.c. shunt motor has armature circuit resistance of 0.5 Ω and a field circuit resistance of 125 Ω. It drives a load at 1000 r.p.m. and takes 30 A. The field circuit resistance is then slowly increased to 150 Ω. If the flux and field current can be assumed to be proportional and if the load torque remains constant, calculate the final speed and armature current. [1186 r.p.m. 33.6 A]

7. A 250 V, shunt motor with an armature resistance of 0.5 Ω and a shunt field resistance of 250 Ω drives a load the torque of which remains constant. The motor draws from the supply a line current of 21 A when the speed is 600 r.p.m. If the speed is to be raised to 800 r.p.m., what change must be affected in the shunt field resistance? Assume that the magnetization curve of the motor is a straight line.

8. A 240 V, d.c. shunt motor runs at 800 r.p.m. with no extra resistance in the field or armature circuit, on no-load. Determine the resistance to be placed in series with the field so that the motor may run at 950 r.p.m. when taking an armature current of 20 A. Field resistance = 160 Ω. Armature resistance = 0.4 Ω. It may be assumed that flux per pole is proportional to field current. [33.6 Ω]

9. A shunt-wound motor has a field resistance of 400 Ω and an armature resistance of 0.1 Ω and runs off 240 V supply. The armature current is 60 A and the motor speed is 900 r.p.m.; Assuming a straight line magnetization curve, calculate (a) the additional resistance in the field to increase the speed to 1000 r.p.m. for the same armature current and (b) the speed with the original field current of 200 A. [(a) 44.4 Ω (b) 842.5 r.p.m.]

10. A 230 V d.c. shunt motor has an armature resistance of 0.5 Ω and a field resistance of 76 2/3 Ω. The motor draws a line current of 13 A while running light at 1000 r.p.m. At a certain load, the field circuit resistance is increased by 38 2/3 Ω. What is the new speed of the motor if the line current at this load is 42 A? [1400 r.p.m.] (Electrical Engg.; Grad I.E.T.E. Dec. 1986)

11. A 250 V d.c. shunt motor runs at 1000 r.p.m. and takes an armature current of 25 amp. Its armature resistance is 0.40 ohm. Calculate the speed with increased load with the armature current of 50 amp. Assume that the increased load results into flux-weakening by 3%, with respect to the flux in previous loading condition. (Nagpur Univ., April 1996)

**Hint:**

(i) First Loading condition:

\[ E_{b1} = 250 - 25 \times 0.40 = K_1 \times 1000 \]

(ii) Second Loading condition:

\[ E_{b2} = 250 - 50 \times 0.40 = 230 K_1 \times (0.97 \phi) \times N_2 \]. This gives \( N_2 \). [988 r.p.m.]

**Armature or Rheostatic Control Method**

This method is used when speeds below the no-load speed are required. As the supply voltage is normally constant, the voltage across the armature is varied by inserting a variable rheostat or resistance (called controller resistance) in series with the armature circuit as shown in Fig. 30.4 (a). As controller resistance is increased, p.d. across the armature is decreased, thereby decreasing the armature speed. For a load constant torque, speed is approximately proportional to the p.d. across the armature. From the speed/armature current characteristic [Fig. 30.4 (b)], it is seen that greater the resistance in the armature circuit, greater is the fall in the speed.
Let
\[ I_{a1} = \text{armature current in the first case} \]
\[ I_{a2} = \text{armature current in the second case} \]
(If \( I_{a1} = I_{a2} \), then the load is of constant torque.)
\[ N_1, N_2 = \text{corresponding speeds,} \ V = \text{supply voltage} \]

Then
\[ N_1 \propto \frac{V - I_{a1} R_a}{E_{b1}} \]

Let some controller resistance of value \( R \) be added to the armature circuit resistance so that its value becomes \((R + R_a) = R_t\).

Then
\[ N_2 \propto \frac{V - I_{a2} R_t}{E_{b2}} \]

\[ \therefore \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \]

(In fact, it is a simplified form of relation given in Art. 27.9 because here \( \Phi_1 = \Phi_2 \).)

Considering no-load speed, we have
\[ \frac{N}{N_0} = \frac{V - I_a R_t}{V - I_{a0} R_a} \]

Neglecting \( I_{a0} R_a \) with respect to \( V \), we get
\[ N = N_0 \left(1 - \frac{I_a R_t}{V}\right) \]

It is seen that for a given resistance \( R_t \), the speed is a linear function of armature current \( I_a \) as shown in Fig. 30.5 (a).

The load current for which the speed would be zero is found by putting \( N = 0 \) in the above relation.

\[ 0 = N_0 \left(1 - \frac{I_a R_t}{V}\right) \text{ or } I_a = \frac{V}{R_t} \]

This is the maximum current and is known as **stalling** current.

As will be shown in Art. 30.5 (a), this method is very wasteful, expensive and unsuitable for rapidly changing loads because for a given value of \( R_t \), speed will change with load.

A more stable operation can be obtained by using a divertor across the armature in addition to armature control resistance (Fig. 30.5 (b)). Now, the changes in armature current (due to changes in the load torque) will not be so effective in changing the p.d. across the armature (and hence the armature speed).

**Example 30.18.** A 200 V d.c. shunt motor running at 1000 r.p.m. takes an armature current of 17.5 A. It is required to reduce the speed to 600 r.p.m. What must be the value of resistance to be inserted in the armature circuit if the original armature resistance is 0.4Ω? Take armature current to be constant during this process.

(Elect. Engg. I Nagpur Univ. 1993)

**Solution.**
\[ N_1 = 1000 \text{ r.p.m.} ; E_{b1} = 200 - 17.5 \times 0.4 = 193 \text{ V} \]
\[ R_t = \text{total arm. circuit resistance} ; N_2 = 600 \text{ r.p.m.} ; E_{b2} = (200 - 17.5 R_t) \]

Since \( I_{sh} \) remains constant,
\[ \Phi_1 = \Phi_2 \]

\[ \therefore \frac{600}{1000} = \frac{(200 - 17.5 R_t)}{193} \text{ or } R_t = 4.8 \text{ \Omega} \]

\[ \therefore \text{Additional resistance reqd.} \ R = R_t - R_a = 4.8 - 0.4 = 4.4 \Omega. \]

It may be noted that brush voltage drop has not been considered.

**Example 30.19.** A 500 V d.c. shunt motor has armature and field resistances of 1.2 \( \Omega \) and 500 \( \Omega \) respectively. When running on no-load, the current taken is 4 A and the speed is 1000 r.p.m. Calculate the speed when motor is fully loaded and the total current drawn from the supply is 26 A. Estimate the speed at this load if (a) a resistance of 2.3 \( \Omega \) is connected in series with the armature and (b) the shunt field current is reduced by 15%. (Electrical Engg. I, Sd. Patel Univ. 1985)
Solution. \[ I_{sh} = \frac{500}{500} = 1 \text{ A}; \quad I_{a1} = 4 - 1 = 3 \text{ A} \]
\[ E_{b1} = 500 - (3 \times 1.2) = 496.4 \text{ V} \quad I_{a2} = 26 - 1 = 25 \text{ A} \]
\[ E_{b2} = 500 - (25 \times 1.2) = 470 \text{ V} \quad \because \quad \frac{N_2}{1000} = \frac{470}{496.4} ; \quad N_2 = 947 \text{ r.p.m.} \]

(a) In this case, total armature circuit resistance is \[ 1.2 + 2.3 = 3.5 \Omega \]
\[ \therefore \quad E_{b2} = 500 - (25 \times 3.5) = 412.5 \text{ V} \quad \therefore \quad \frac{N_2}{1000} = \frac{412.5}{496.4} \times \frac{1}{0.85} ; \quad N_2 = 831 \text{ r.p.m.} \]

(b) When shunt field is reduced by 15\%, \( \Phi_2 = 0.85 \Phi_1 \) assuming straight magnetisation curve.
\[ \frac{N_3}{1000} = \frac{412.5}{496.4} \times \frac{1}{0.85} ; \quad N_2 = 977.6 \text{ r.p.m.} \]

Example 30.20. A 250-V shunt motor (Fig. 30.6) has an armature current of 20 A when running at 1000 r.p.m. against full load torque. The armature resistance is 0.5 \( \Omega \). What resistance must be inserted in series with the armature to reduce the speed to 500 r.p.m. at the same torque and what will be the speed if the load torque is halved with this resistance in the circuit? Assume the flux to remain constant throughout and neglect brush contact drop.

(Elect. Machines AMIE Sec. B Summer 1991)

Solution. \[ E_{b1} = V - I_{a1} R_a = 250 - 20 \times 0.5 = 240 \text{ V} \]

Let \( R_i \) to be total resistance in the armature circuit i.e. \( R_i = R_a + R \), where \( R \) is the additional resistance.

\[ \therefore \quad E_{b2} = V - I_{a2} R_i = 250 - 20 R_i \]

It should be noted that \( I_{a1} = I_{a2} = 20 \text{ A} \) because torque remains the same and \( \Phi_1 = \Phi_2 \) in both cases.

\[ \therefore \quad \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \text{ or } \frac{500}{1000} = \frac{250 - 20 R_i}{240} \]

\[ \therefore \quad R_i = 6.5 \Omega ; \quad \text{hence, } R = 6.5 - 0.5 = 6 \Omega \]

Since the load is halved, armature current is also halved because flux remains constant. Hence, \( I_{a3} = 10 \text{ A} \).

\[ \therefore \quad \frac{N_3}{1000} = \frac{250 - 10 \times 6.5}{240} \text{ or } N_3 = 771 \text{ r.p.m.} \]

Example 30.21. A 250-V shunt motor with armature resistance of 0.5 ohm runs at 600 r.p.m. on full-load and takes an armature current of 20 A. If resistance of 1.0 ohm is placed in the armature circuit, find the speed at (i) full-load torque (ii) half full-load torque.

(Electrical Machines-II, Punjab Univ. May 1991)

Solution. Since flux remains constant, the speed formula becomes \[ \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \]

(i) In the first case, full-load torque is developed.
\[ N_1 = 600 \text{ r.p.m.}; \quad E_{b1} = V - I_{a1} R_{a1} = 250 - 20 \times 0.5 = 240 \text{ V} \]

Now, \[ T \propto \Phi I_a \propto I_a \] \( (\therefore \Phi \text{ is constant}) \)

\[ \therefore \quad \frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}} \quad \text{Since } T_2 = T_1 \quad I_{a2} = I_{a1} = 20 \text{ A} \]

\[ E_{b2} = V - I_{a2} R_{a2} = 250 - 20 \times 1.5 = 220 \text{ V} \]

\[ N_2 = \frac{N_2}{600} = \frac{220}{240} ; \quad N_2 = \frac{600 \times 220}{240} = 550 \text{ r.p.m.} \]

(ii) In this case, the torque developed is half the full-load torque.
\[
\frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}} \quad \text{or} \quad \frac{T_1/T_2}{I_{a1}} = \frac{I_{a2}}{20} ; \quad I_{a2} = 10 \, \text{A} ; \quad E_{b2} = 250 - 10 \times 1.5 = 235 \, \text{V} \\
\frac{N_2}{600} = \frac{235}{240} ; \quad N_2 = 600 \times \frac{235}{240} = 587.5 \, \text{r.p.m.}
\]

**Example 30.22.** A 220 V shunt motor with an armature resistance of 0.5 ohm is excited to give constant main field. At full load the motor runs of 500 rev. per minute and takes an armature current of 30 A. If a resistance of 1.0 ohm is placed in the armature circuit, find the speed at (a) full-load torque (b) double full-load torque. (Elect. Machines-I, Nagpur Univ. 1993)

**Solution.** Since flux remains constant, the speed formula becomes \( N_2/N_1 = E_{b2}/E_{b1} \).

(a) **Full-load torque**

With no additional resistance in the armature circuit,

\[ N_1 = 500 \, \text{r.p.m.} ; \quad I_{a1} = 30 \, \text{A} ; \quad E_{b1} = 220 - 30 \times 0.5 = 205 \, \text{V} \]

Now, \( T \propto I_a \) (since \( \Phi \) is constant.) \[ \therefore \frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}} \]

Since \( T_2 = T_1 \); \( I_{a2} = I_{a1} = 30 \, \text{A} \)

When additional resistance of 1 \( \Omega \) is introduced in the armature circuit,

\[ E_{b2} = 220 - 30 (1 + 0.5) = 175 \, \text{V} ; \quad N_2 = \frac{N_2}{500} = \frac{175}{205} ; \quad N_2 = 427 \, \text{r.p.m.} \]

(b) **Double Full-load Torque**

\[ \frac{T_2}{T_1} = \frac{I_{a2}}{I_{a1}} \quad \text{or} \quad \frac{2T_1}{I_{a1}} = \frac{I_{a2}}{30} ; \quad I_{a2} = 60 \, \text{A} \]

\[ \therefore \quad E_{b2} = 220 - 60 (1 + 0.5) = 130 \, \text{V} \]

\[ \therefore \quad \frac{N_2}{500} = \frac{130}{205} ; \quad N_2 = 317 \, \text{r.p.m.} \]

**Example 30.23.** The speed of a 50 h.p (37.3 kW) series motor working on 500 V supply is 750 r.p.m. at full-load and 90 per cent efficiency. If the load torque is made 350 N-m and a 5 ohm resistance is connected in series with the machine, calculate the speed at which the machine will run. Assume the magnetic circuit to be unsaturated and the armature and field resistance to be 0.5 ohm. (Electrical Machinery I, Madras Univ. 1986)

**Solution.** Load torque in the first case is given by

\[ T_1 = 37,300/2\pi \times (750/60) = 474.6 \, \text{N-m} \]

Input current, \( I_{a1} = 37,300/0.9 \times 500 = 82.9 \, \text{A} \)

Now,

\[ T_2 = 250 \, \text{N-m} ; \quad I_{a2} = ? \]

In a series motor, before magnetic saturation,

\[ T \propto \Phi I_a \propto I_a^2 \quad \therefore T_1 \propto I_{a1}^2 \text{ and } T_2 \propto I_{a2}^2 \]

\[ \therefore \quad \frac{I_{a2}^2}{I_{a1}^2} = \frac{T_2}{T_1} \quad \therefore I_{a2} = \sqrt{82.9 \times 250/474.6} = 60.2 \, \text{A} \]

Now,

\[ E_{b1} = 500 - (82.9 \times 0.5) = 458.5 \, \text{V} \]

\[ E_{b2} = 500 - 60.2 (5 + 0.5) = 168.9 \, \text{V} \]

Using \( \frac{N_2}{N_1} = \frac{E_{b2} \times I_{a2}}{E_{b1} \times I_{a1}} \), we get

\[ N_2 = \frac{168.9 \times 82.9}{458.5 \times 60.2} \quad \therefore N_2 = 381 \, \text{r.p.m.} \]

**Example 30.24.** A 7.46 kW, 220 V, 900 r.p.m. shunt motor has a full-load efficiency of 88 per cent, an armature resistance of 0.08 \( \Omega \) and shunt field current of 2 A. If the speed of this motor is
reduced to 450 r.p.m. by inserting a resistance in the armature circuit, the load torque remaining constant, find the motor output, the armature current, the external resistance and the overall efficiency.

(Elect. Machines, Nagpur Univ. 1993)

**Solution.** Full-load motor input current \( I = \frac{7460}{220} \times 0.88 = 38.5 \text{ A} \)

\[ I_{a1} = 38.5 - 2 = 36.5 \text{ A} \]

Now, \( T \propto \Phi I_a \) Since flux remains constant.

\[ T \propto I_a \implies T_{a1} \propto I_{a1} \text{ and } T_{a2} \propto I_{a2} \text{ or } \frac{T_{a2}}{T_{a1}} = \frac{I_{a2}}{I_{a1}} \]

It is given that \( T_{a1} = T_{a2} \); hence \( I_{a1} = I_{a2} = 36.5 \text{ A} \)

\[ E_{b1} = 220 - (36.5 \times 0.08) = 217.1 \Omega \]

\[ E_{b2} = 220 - 36.5 R_l; N_1 = 900 \text{ r.p.m.}; N_2 = 450 \text{ r.p.m.} \]

Now,

\[ \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} = \frac{E_{b2}}{E_{b1}} \]

\[ \implies \frac{450}{900} = \frac{200 - 36.5 R_l}{217.1}; R_l = 3.05 \Omega \]

\[ \therefore \text{ External resistance } R = 3.05 - 0.08 = 2.97 \Omega \]

For calculating the motor output, it will be assumed that all losses except copper losses vary directly with speed.

Since motor speed is halved, stray losses are also halved in the second case. Let us find their value.

In the first case, motor input = \( 200 \times 38.5 = 8,470 \text{ W} \); Motor output = 7,460 W

Total Cu losses + stray losses = 8470 - 7460 = 1010 W

Arm. Cu loss = \( I_{a1}^2 R_a = 36.5^2 \times 0.08 = 107 \text{ W} \); Field Cu loss = \( 220 \times 2 = 440 \text{ W} \)

Total Cu loss = 107 + 440 = 547 W \[ \therefore \text{ Stray losses in first case = 1010 - 547 = 463 W} \]

Stray losses in the second case = \( 463 \times \frac{450}{900} = 231 \text{ W} \)

Field Cu loss = 440 W, as before; Arm. Cu loss = \( 36.5^2 \times 3.05 = 4,064 \text{ W} \)

Total losses in the 2nd case = 231 + 440 + 4,064 = 4,735 W

Input = 8,470 W \[ \therefore \text{ Output in the second case = 8,470} - 4,735 = 3,735 \text{ W} \]

\[ \therefore \text{ Overall } \eta = \frac{3,735}{8,470} = 0.441 \text{ or } 44.1 \text{ per cent*} \]

**Example 30.25.** A 240 V shunt motor has an armature current of 15 A when running at 800 r.p.m. against F.L. torque. The arm. resistance is 0.6 ohms. What resistance must be inserted in series with the armature to reduce the speed to 400 r.p.m., at the same torque?

What will be the speed if the load torque is halved with this resistance in the circuit? Assume the flux to remain constant throughout.

(Elect. Machines-I Nagpur Univ. 1993)

**Solution.** Here,

\[ N_1 = 800 \text{ r.p.m.}, E_{b1} = 240 - 15 \times 0.6 = 231 \text{ V} \]

Flux remaining constant, \( T \propto I_a \). Since torque is the same in both cases, \( I_{a2} = I_{a1} = 15 \text{ A} \). Let \( R \) be the additional resistance inserted in series with the armature. \( E_{b2} = 240 - 15 (R + 0.6) \); \( N_2 = 400 \text{ r.p.m.} \)

\[ \frac{400}{800} = \frac{240 - 15 (R + 0.6)}{231}; R = 7.7 \Omega \]

* It may be noted that efficiency is reduced almost in the ratio of the two speeds.
When load torque is halved:

With constant flux when load torque is halved, \( I_a \) is also halved. Hence, \( I_{a3} = I_{a1}/2 = 15/2 = 7.5 \) A

\[
E_{b3} = 240 - 7.5 (7.7 + 0.6) = 177.75 V ; \quad N_3 = ?
\]

\[
\frac{N_3}{N_1} = \frac{E_{b3}}{E_{b4}} \quad \text{or} \quad \frac{N_3}{800} = \frac{177.75}{231} \quad ; \quad N_3 = 614.7 \text{ r.p.m.}
\]

**Example 30.26.** (a) A 400 V shunt connected d.c. motor takes a total current of 3.5 A on no load and 59.5 A at full load. The field circuit resistance is 267 ohms and the armature circuit resistance is 0.2 ohms (excluding brushes where the drop may be taken as 2 V). If the armature reaction effect at 'full-load' weakens the flux per pole by 2 percentage change in speed from no-load to full-load.

(b) What resistant must be placed in series with the armature in the machine of (a) if the full-load speed is to be reduced by 50 per cent with the gross torque remaining constant? Assume no change in the flux.

*(Electrical Machines, AMIE Sec. B, 1989)*

**Solution.** (a) Shunt current \( I_{sh} = 400/267 = 1.5 \) A. At no load, \( I_{a1} = 3.5 - 1.5 = 2 \) A, \( E_{b1} = V - I_{a1} R_{ai} - \) brush drop = 400 - 2 x 0.2 - 2 = 397.6 V. On full-load, \( I_{a2} = 59.5 - 1.5 = 58 \) A, \( E_{b2} = 400 - 58 \times 0.2 - 2 = 386.4 \) V.

\[
\frac{E_{b1}}{E_{b2}} = \frac{\Phi_1 N_1}{\Phi_2 N_2} \quad \text{or} \quad \frac{397.6}{386.4} = \frac{\Phi_1 N_1}{0.98 \Phi_1 N_2} \quad ; \quad N_1 = 1.0084
\]

\[
\% \text{ change in speed} = \frac{N_1 - N_2}{N_1} \times 100
\]

\[
= \left(1 - \frac{1}{1.0084}\right) \times 100 = 0.833
\]

(b) Since torque remains the same, \( I_a \) remains the same, hence \( I_{a3} = I_{a2} \). Let \( R \) be the resistance connected in series with the armature.

\[
E_{b3} = V - I_{a2} (R_a + R) - \text{brush drop}
\]

\[
= 400 - 58 (0.2 + R) - 2 = 386.4 - 58 R
\]

\[
\frac{E_{b2}}{E_{b3}} = \frac{\Phi_2 N_2}{\Phi_3 N_3} = \frac{N_2}{N_3}
\]

\[
\frac{386.4}{386.4 - 58 R} = \frac{1}{0.5} \quad ; \quad R = 3.338 \Omega
\]

**Example 30.27.** A d.c. shunt drives a centrifugal pump whose torque varies as the square of the speed. The motor is fed from a 200 V supply and takes 50 A when running at 1000 r.p.m. What resistance must be inserted in the armature circuit in order to reduce the speed to 800 r.p.m.? The armature and field resistance of the motor are 0.1 \( \Omega \) and 100 \( \Omega \) respectively.

*(Elect. Machines, Allahabad Univ. 1992)*

**Solution.** In general, \( T \propto \Phi I_a \).

For shunt motors whose excitation is constant,

\[
T \propto I_a \propto N^2 \quad \text{as given.}
\]

\[
\therefore \quad I_a \propto N^2 \quad \text{. Now} \quad I_{sh} = 200/100 = 2 \text{ A} \quad \therefore \quad I_{a1} = 50 - 2 = 48 \text{ A}
\]

Let

\[
I_{a2} = \text{new armature current at 800 r.p.m.}
\]

then

\[
48 \propto N_1^2 \propto 1000^2 \quad \text{and} \quad I_{a2} \propto N_2^2 \propto 800^2
\]

\[
\therefore \quad \frac{I_{a2}}{48} = \left(\frac{800}{1000}\right)^2 = 0.8^2 \quad \therefore \quad I_{a2} = 48 \times 0.64 = 30.72 \text{ A}
\]
\[
E_{b1} = 200 - (48 \times 0.1) = 195.2 \text{ V} \; ; \; E_{b2} = (200 - 30.72 R_h) \text{ V}
\]

Now,
\[
\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} = \frac{800}{1000} = \frac{200 - 30.72 R_h}{195.2}, \quad R_h = 1.42 \Omega
\]

Additional resistance = 1.42 - 0.1 = 1.32 \Omega

**Example 30.28.** A 250 V, 50 h.p. (373 kW) d.c. shunt motor has an efficiency of 90% when running at 1,000 r.p.m. on full-load. The armature and field resistances are 0.1 \Omega and 115 \Omega respectively. Find

(a) the net and developed torque on full-load.

(b) the starting resistance to have the line start current equal to 1.5 times the full-load current.

(c) the torque developed at starting. (Elect. Machinery-I, Kerala Univ. 1987)

**Solution.** (a) \( T_{sh} = 9.55 \times 37,300/1000 = 356.2 \text{ N-m} \)

Input current \( I_a = \frac{37,300}{250 \times 0.9} = 165.8 \text{ A} \; ; \; I_{sh} = \frac{250}{125} = 2 \text{ A} \)

\[ I_a = 165.8 - 2 = 163.8 \text{ A} \; ; \; E_b = 250 - (163.8 \times 0.1) = 233.6 \text{ V} \]

\[ T_a = 9.55 \times \frac{233.6 \times 163.8}{1000} = 365.4 \text{ N-m} \]

(b) F.L. input line \( I = 165.8 \text{ A} \); Permissible input \( I = 165.8 \times 1.5 = 248.7 \text{ A} \)

Permissible armature current = 248.7 - 2 = 246.7 A

Total armature resistance = 250/246.7 = 1.014 \Omega

\[ \therefore \text{Starting resistance required} = 1.014 - 0.1 = 0.914 \Omega \]

(c) Torque developed with 1.5 times the F.L. current would be practically 1.5 times the F.L. torque.

\[ i.e. \quad 1.5 \times 365.4 = 548.1 \text{ N-m}. \]

**Example 30.29.** A 200 V shunt motor with a shunt resistance of 40 \Omega and armature resistance of 0.02 \Omega takes a current of 55 A and runs at 595 r.p.m. when there is a resistance of 0.58 \Omega in series with armature. Torque remaining the same, what change should be made in the armature circuit resistance to raise the speed to 630 r.p.m.? Also find

(i) At what speed will the motor run if the load torque is reduced such that armature current is 15 A.

(ii) Now, suppose that a divertor of resistance 5 \Omega is connected across the armature and series resistance is so adjusted that motor speed is again 595 r.p.m., when armature current is 50 A. What is the value of this series resistance? Also, find the speed when motor current falls of 15 A again.

**Solution.** The circuit is shown in Fig. 30.7.

![Fig. 30.7](image-url)

![Fig. 30.8](image-url)
\[ I_{sh} = \frac{200}{40} = 5 \text{ A} \quad \therefore \quad I_{a1} = 55 - 5 = 50 \text{ A} \]

Armature circuit resistance = \(0.58 + 0.02 = 0.6 \, \Omega\)

\[ \therefore \quad E_{b1} = 200 - (50 \times 0.6) = 170 \text{ V} \]

Since torque is the same in both cases, \(I_{a1} \Phi_1 = I_{a2} \Phi_2\)

Moreover, \(\Phi_1 = \Phi_2\) \quad \therefore \quad \Phi_1 = \Phi_2\)

Now \(E_{b1} = 170 \text{ V}, \quad N_1 = 595 \text{ r.p.m.}, \quad N_2 = 630 \text{ r.p.m.}, \quad E_{b2} = ?\)

Using \(\frac{N_2}{N_1} = E_{b2} \div E_{b1}\)

we get \(E_{b2} = 170 \times (630/595) = 180 \text{ V}\)

Let \(R_2\) be the new value of armature circuit resistance for raising the speed from 595 r.p.m. to 630 r.p.m.

\[ 180 = 200 - 50 R_2 \quad \therefore \quad R = 0.4 \, \Omega \]

Hence, armature circuit resistance should be reduced by \(0.6 - 0.4 = 0.2 \, \Omega\).

(i) We have seen above that

\[ I_{a1} = 50 \text{ A}, \quad E_{b1} = 170 \text{ V}, \quad N_1 = 595 \text{ r.p.m.} \]

If \(I_{a2} = 15 \text{ A}, \quad E_{b2} = 200 - (15 \times 0.6) = 191 \text{ V}\)

\[ \therefore \quad \frac{N_2}{595} = \frac{191}{170} \quad \therefore \quad N_2 = 668.5 \text{ r.p.m.} \]

(ii) When armature diverter is used (Fig. 30.8).

Let \(R\) be the new value of series resistance

\[ E_{b3} = 200 - IR - (50 \times 0.02) = 199 - IR \]

Since speed is 595 r.p.m., \(E_{b3}\) must be equal to 170 V

\[ 170 = 199 - IR \quad \therefore \quad IR = 29 \text{ V}; \quad \text{P.D. across diverter} = 200 - 29 = 171 \text{ V} \]

Current through diverter \(I_d\) = \(171/5 = 34.2 \text{ A} \quad \therefore \quad I = 50 + 34.2 = 84.2 \text{ A} \)

As \(IR = 29 \text{ V} \quad \therefore \quad R = 29/84.2 = 0.344 \text{ W} \)

When \(I_a = 15 \text{ A}, \) then \(I_d = (I - 15) \text{ A} \)

P.D. across diverter = \(5 (I - 15) = 200 - 0.344 \text{ I} \quad \therefore \quad I = 51.46 \text{ A} \)

\[ E_{b4} = 200 - 0.344 I - (15 \times 0.02) \]

\[ = 200 - (0.344 \times 51.46) - 0.3 = 182 \text{ V} \]

\[ \therefore \quad \frac{N_4}{N_1} = \frac{E_{b4}}{E_{b1}} \quad \text{or} \quad \frac{N_4}{595} = \frac{182}{170} \quad \therefore \quad N_4 = 637 \text{ r.p.m.} \]

The effect of armature diverter is obvious. The speed without diverter is 668.5 r.p.m. and with armature diverter, it is 637 r.p.m.

(iii) Voltage Control Method

(a) Multiple Voltage Control

In this method, the shunt field of the motor is connected permanently to a fixed exciting voltage, but the armature is supplied with different voltages by connecting it across one of the several different voltages by means of suitable switchgear. The armature speed will be approximately proportional to these different voltages. The intermediate speeds can be obtained by adjusting the shunt field regulator. The method is not much used, however.

(b) Ward-Leonard System

This system is used where an unusually wide (upto 10 : 1) and very sensitive speed control is required as for colliery winders, electric excavators, elevators and the main drives in steel mills and blooming and paper mills. The arrangement is illustrated in Fig. 30.9.

\(M_1\) is the main motor whose speed control is required. The field of this motor is permanently connected across the d.c. supply lines. By applying a variable voltage across its armature, any desired
speed can be obtained. This variable voltage is supplied by a motor-generator set which consists of either a d.c. or an a.c. motor $M$, directly coupled to generator $G$.

![Diagram of motor-generator set](image)

Fig. 30.9

The motor $M_2$ runs at an approximately constant speed. The output voltage of $G$ is directly fed to the main motor $M_1$. The voltage of the generator can be varied from zero up to its maximum value by means of its field regulator. By reversing the direction of the field current of $G$ by means of the reversing switch $RS$, generated voltage can be reversed and hence the direction of rotation of $M_1$. It should be remembered that motor generator set always runs in the same direction.

Despite the fact that capital outlay involved in this system is high because (i) a large output machine must be used for the motor generator set and (ii) that two extra machines are employed, still it is used extensively for elevators, hoist control and for main drive in steel mills where motor of ratings 750 kW to 3750 kW are required. The reason for this is that the almost unlimited speed control in either direction of rotation can be achieved entirely by field control of the generator and the resultant economies in steel production outweigh the extra expenditure on the motor generator set.

A modification of the Ward-Leonard system is known as Ward-Leonard-Ilgner system which uses a smaller motor-generator set with the addition of a flywheel whose function is to reduce fluctuations in the power demand from the supply circuit. When main motor $M_1$ becomes suddenly overloaded, the driving motor $M_2$ of the motor generator set slows down, thus allowing the inertia of the flywheel to supply a part of the overload. However, when the load is suddenly thrown off the main motor $M_1$, then $M_2$ speeds up, thereby again storing energy in the flywheel.

When the Ilgner system is driven by means of an a.c. motor (whether induction or synchronous) another refinement in the form of a ‘slip regulator’ can be usefully employed, thus giving an additional control.

The chief disadvantage of this system is its low overall efficiency especially at light loads. But as said earlier, it has the outstanding merit of giving wide speed control from maximum in one direction through zero to the maximum in the opposite direction and of giving a smooth acceleration.

**Example 30.30.** The O.C.C. of the generator of a Ward-Leonard set is

<table>
<thead>
<tr>
<th>Field amps</th>
<th>1.4</th>
<th>2.2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Armature volts</td>
<td>212</td>
<td>320</td>
<td>397</td>
<td>472</td>
<td>522</td>
<td>560</td>
<td>586</td>
<td>609</td>
</tr>
</tbody>
</table>

The generator is connected to a shunt motor, the field of which is separately-excited at 550 V. If the speed of motor is 300 r.p.m. at 550 V, when giving 485 kW at 95.5% efficiency, determine the excitation of the generator to give a speed of 180 r.p.m. at the same torque. Resistance of the motor
armature circuit = 0.01 \, \Omega, \text{ resistance of the motor field} = 60 \, \Omega, \text{ resistance of generator armature circuit} = 0.01 \, \Omega. \text{ Ignore the effect of armature reaction and variation of the core factor and the windage losses of the motor.}

\textbf{Solution.}

\begin{align*}
\text{Motor input} &= 485 \times 10^3 / 0.955 = 509,300 \, \text{W} \\
\text{Motor to motor field} &= 550/60 = 55/6 \, \text{A} \\
\text{Input to motor field} &= 550 \times 55/6 = 5,040 \, \text{W} \\
\therefore \text{Motor armature input} &= 509,300 - 5,040 = 504,260 \, \text{W} \\
\therefore \text{Armature current} &= 504,260/550 = 917 \, \text{A} \\
\text{Back e.m.f.} \ E_{b1} \text{ at 300 r.p.m.} &= 550 - (917 \times 0.01) = 540.83 \, \text{V} \\
\text{Back e.m.f.} \ E_{b2} \text{ at 180 r.p.m.} &= 540.83 \times 180/300 = 324.5 \, \text{V} \\
\text{Since torque is the same, the armature current of the main motor is also the same i.e. 917 A because its excitation is independent of its speed.} \\
\therefore \ V &= 324.5 + (917 \times 0.01) = 333.67 \, \text{V} \\
\text{Generated e.m.f.} &= V + I_a R_a \\
333.67 + (917 \times 0.011) &= 343.77 \, \text{V}. \\
\text{If O.C.C. is plotted from the above given data, then it would be found that the excitation required to give 343.77 V is 2.42 A.} \\
\therefore \text{Generator exciting current} &= 2.42 \, \text{A}
\end{align*}

\textbf{30.3. Speed Control of Series Motors}

1. \textbf{Flux Control Method}

Variations in the flux of a series motor can be brought about in any one of the following ways:

\textit{(a) Field Divertors}

The series winding are shunted by a variable resistance known as field divertor (Fig. 30.10). Any desired amount of current can be passed through the divertor by adjusting its resistance. Hence the flux can be decreased and consequently, the speed of the motor increased.

\textit{(b) Armature Divertor}

A divertor across the armature can be used for giving speeds lower than the normal speed (Fig. 30.11). For a \textit{given constant load torque}, if \( I_a \) is reduced due to armature divertor, the \( \Phi \) must increase.

\( \therefore \ T_a \propto \Phi I_a \). This results in an increase in current taken from the supply (which increases the flux and a fall in speed \( N \propto I/\Phi \)). The variation in speed can be controlled by varying the divertor resistance.
(c) Trapped Field Control Field
This method is often used in electric traction and is shown in Fig. 30.12.
The number of series filed turns in the circuit can be changed at will as shown. With full field,
the motor runs at its minimum speed which can be raised in steps by cutting out some of the series
turns.

(d) Paralleling Field coils
In this method, used for fan motors, several speeds can be obtained by regrouping the field coils
as shown in Fig. 30.13. It is seen that for a 4-pole motor, three speeds can be obtained easily. (Ex.30.35)

2. Variable Resistance in Series with Motor
By increasing the resistance in series with the armature (Fig. 30.14) the voltage applied across
the armature terminals can be decreased.

With reduced voltage across the armature, the speed is reduced. However, it will be noted that
since full motor current passes through this resistance, there is a considerable loss of power in it.

Example 30.31. A d.c. series motor drives a load the torque of which varies as the square of the
speed. The motor takes a current of 15 A when the speed is 600 r.p.m. Calculate the speed and the
current when the motor field winding is shunted by a divertor of the same resistance as that of the
field winding. Mention the assumptions made, if any. (Elect. Machines, AMIE Sec B, 1993)

Solution.
\[ T_{a1} \propto N_1^2, \quad T_{a2} \propto N_2^2 \]
\[ T_{a1} \propto \Phi_1 I_{a1}, \quad T_{a2} \propto \Phi_2 I_{a2}, \quad (I_{a2}/2) I_{a2} \propto I_{a2}/2 \]
\[ \therefore \quad \frac{N_2^2}{N_1^2} = \frac{I_{a2}^2/2}{I_{a1}^2} \quad \text{or} \quad \frac{N_2}{N_1} = \frac{I_{a2}}{\sqrt{2} I_{a1}} \quad \ldots \text{(i)} \]
Now,
\[ \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \Phi_1 / \Phi_2 \]

If we neglect the armature and series winding drops as well as brush drop, then 
\[ E_{b1} = E_{b2} = V \]

\[ \frac{N_2}{N_1} = \frac{\Phi_1}{\Phi_2} = \frac{I_{al}}{I_{a2}} = \frac{2 I_{al}}{2 I_{a2}} \]

\[ \text{or} \quad I_{a2} = 2 \sqrt{2} \quad I_{al} = 2 \sqrt{2} \times 15 \quad \text{or} \quad I_{a2} = 25.2 \, \text{A} \]

From (ii), we get,
\[ N_2 = 600 \times 2 \times 15 / 252 = 714 \, \text{r.p.m.} \]

**Example 30.32.** A 2-pole series motor runs at 707 r.p.m. when taking 100 A at 85 V and with the field coils in series. The resistance of each field coil is 0.03 Ω and that of the armature 0.04 Ω. If the field coils are connected in parallel and load torque remains constant, find (a) speed (b) the additional resistance to be inserted in series with the motor to restore the speed to 707 r.p.m.

**Solution.** Total armature circuit resistance = 0.04 + (2 × 0.03) = 0.1 Ω

\[ I_{a1} = 100 \, \text{A} ; \quad E_{b1} = 85 - (100 \times 0.1) = 75 \, \text{V} \]

When series field windings are placed in parallel, the current through each is half the armature current.

If \[ I_{a2} = \text{new armature current} \] ; then \( \Phi_2 \propto I_{a2}/2 \).

As torque is the same in the two cases,

\[ \Phi_1 I_{a1} = \Phi_2 I_{a2} \quad \text{or} \quad I_{a1} = \frac{I_{a2}}{2} \times \frac{I_{a2}}{2} \]

\[ 100^2 = I_{a2}^2 \quad \therefore \quad I_{a2} = 100 \sqrt{2} = 141.4 \, \text{A} \]

In this case, series field resistance = 0.03/2 = 0.015 Ω

\[ E_{b2} = 85 - 141.4 (0.04 + 0.015) = 77.22 \, \text{V} \]

\[ \frac{N_2}{707} = \frac{77.22 \times 100}{75 \times 141.4 / 2} \quad \therefore \quad \Phi_2 \propto I_{a2}/2 \]

(a) \[ \therefore \quad N_2 = 707 \times \frac{77.22}{75} \times \frac{200}{141.4} = 1029 \, \text{r.p.m.} \]

(b) Let the total resistance of series circuit be \( R_t \).

Now, \[ E_{b1} = 77.22 \, \text{V} \]

\[ N_1 = 1029 \, \text{r.p.m.} ; \quad E_{b2} = 85 - 141.4 \, R_t, \quad N_2 = 707 \, \text{r.p.m.} \]

\[ \frac{707}{1029} = \frac{85 - 141.4 \, R_t}{77.22} \quad \therefore \quad R_t = 0.226 \, \Omega \]

**Example 30.33.** A 240 V series motor takes 40 amperes when giving its rated output at 1500 r.p.m. Its resistance is 0.3 ohm. Find what resistance must be added to obtain rated torque (i) at starting (ii) at 1000 r.p.m.

*Elect. Engg., Madras Univ. 1987*

**Solution.** Since torque remains the same in both cases, it is obvious that current drawn by the motor remains constant at 40 A.

\( T_a \propto I_a^2 \)

(i) If \( R \) is the series resistance added, then \( 40 = 240 / (R + 0.3) \) \( \therefore \quad R = 5.7 \, \Omega \)

(ii) Current remaining constant, \( T_a = E_b / N \)

\[ \frac{E_{b1}}{N_1} = \frac{E_{b2}}{N_2} \]

Now,
\[ E_{b1} = 240 - 40 \times 0.3 = 228 \, \text{V} ; \quad N_1 = 1500 \, \text{r.p.m.} \]
\[ E_{b2} = 240 - 40 (R + 0.3) \, \text{V} ; \quad N_2 = 1000 \, \text{r.p.m.} \]
\[ \frac{228}{1500} = \frac{240 - 40(R + 0.3)}{1000} ; \quad R = 1.9 \, \Omega \]

**Example 30.34.** A 4-pole, series-wound fan motor runs normally at 600 r.p.m. on a 250 V d.c. supply taking 20 A. The field coils are connected in series. Estimate the speed and current taken by the motor if the coils are reconnected in two parallel groups of two in series. The load torque increases as the square of the speed. Assume that the flux is directly proportional to the current and ignore losses.

*(Elect. Machines, AMIE, Sec B. 1990)*

**Solution.** When coils are connected in two parallel groups, current through each becomes \( I_{a2}/2 \) where \( I_{a2} \) is the new armature current.

Hence,

\[ \Phi_2 \propto I_{a2}/2 \]

Now

\[ T_a \propto \Phi I_a \propto N^2 \]

--- given

\[ \Phi_1 I_{a1} \propto N_1^2 \quad \text{and} \quad \Phi_2 I_{a2} \propto N_2^2 \]

\[ \frac{N_2}{N_1} = \frac{E_{b2}/\Phi_1}{E_{b1}/\Phi_2} \]

\( \therefore \) \( \Phi_1 I_{a1} \propto N_1^2 \) and \( \Phi_2 I_{a2} \propto N_2^2 \).

\[ \frac{N_2}{N_1} = \frac{\Phi_1 I_{a1}}{\Phi_2 I_{a2}} \]

Since losses are negligible, field coil resistance as well as armature resistance are negligible. It means that armature and series field voltage drops are negligible. Hence, back e.m.f. in each case equals the supply voltage.

\[ \frac{N_2}{N_1} = \frac{E_{b2}/\Phi_1}{E_{b1}/\Phi_2} \]

Putting this value in (i) above, we get

\[ \left( \frac{\Phi_1}{\Phi_2} \right)^2 = \frac{\Phi_2 I_{a2}}{\Phi_1 I_{a1}} \quad \text{or} \quad \frac{I_{a2}}{I_{a1}} = \left( \frac{\Phi_1}{\Phi_2} \right)^3 \]

Now, \( \Phi_1 \propto 20 \) and \( \Phi_2 \propto I_{a2}/2 \) :: \[ \frac{I_{a2}}{I_{a1}} = \left( \frac{20}{I_{a2}/2} \right)^3 \]

\[ I_{a2} = 20 \times 2^{3/4} = 33.64 \, \text{A} \]

From (ii) above, we get

\[ \frac{N_2}{N_1} = \frac{\Phi_1}{\Phi_2} = \frac{I_{a1}}{I_{a2}} = \frac{2 I_{a1}}{I_{a2}} ; \quad N_2 = 600 \times 2 \times 20/33.64 = 714 \, \text{r.p.m.} \]

**Example 30.35.** A d.c. series motor having a resistance of 1 \( \Omega \) drives a fan for which the torque varies as the square of the speed. At 220 V, the set runs at 350 r.p.m. and takes 25 A. The speed is to be raised to 500 r.p.m. by increasing the voltage. Determine the necessary voltage and the corresponding current assuming the field to be unsaturated.

*(Electrical Engg., Banaras Hindu Univ. 1998)*

**Solution.** Since \( \Phi \propto I_a \), hence \( T_a \propto \Phi I_a \propto I_a^2 \). Also \( T_a \propto N^2 \), \( I_a \propto N \) or \( I_{a1} \propto N_1 \) and \( I_{a2} \propto N_2 \).

\[ \frac{I_{a2}}{I_{a1}} = \frac{N_2}{N_1} = \frac{500}{350} ; \quad I_{a2} = 25 \times \frac{500}{350} = \frac{250}{7} \, \text{A} \]

\[ E_{b1} = 220 - 25 \times 1 = 195 \, \text{V} ; \quad E_{b1} = V - (250/7) \times 1, \quad \frac{\Phi_1}{\Phi_2} = \frac{25}{250/7} = \frac{7}{10} \]

Now

\[ \frac{N_2}{N_1} = \frac{E_{b2}/\Phi_1}{E_{b1}/\Phi_2} = \frac{500}{350} = \frac{V - (250/7) \times 1}{195} \times \frac{7}{10} ; \quad V = 433.7 \, \text{V} \]

**Example 30.36.** A d.c. series motor runs at 1000 r.p.m. when taking 20 A at 200 V. Armature resistance is 0.5 \( \Omega \). Series field resistance is 0.2 \( \Omega \). Find the speed for a total current of 20 A when a diverter of 0.2 \( \Omega \) resistance is used across the series field. Flux for a field current of 10 A is 70 per cent of that for 20 A.
Solution. \( E_{b1} = 200 - (0.5 + 0.2) \times 20 = 186 \, V \); \( N_1 = 1000 \, \text{r.p.m.} \)

Since divertor resistance equals series field resistance, the motor current of 20 A is divided equally between the two. Hence, a current of 10 A flows through series field and produces flux which is 70% of that corresponding to 20 A. In other words, \( \Phi_2 = 0.7 \) or \( \Phi_1/\Phi_2 = 1/0.7 \).

Moreover, their combined resistance = \( 0.2/2 = 0.1 \, \Omega \)

Total motor resistance becomes = \( 0.5 + 0.1 = 0.6 \, \Omega \)

\[ E_{b2} = 200 - 0.6 \times 20 = 188 \, V \]; \( N_2 = ? \)

\[ \frac{N_2}{1000} = \frac{188}{186} \times \frac{1}{0.7} \]; \( N_2 = 1444 \, \text{r.p.m.} \)

Example 30.37. A 200 V, d.c. series motor takes 40 A when running at 700 r.p.m. Calculate the speed at which the motor will run and the current taken from the supply if the field is shunted by a resistance equal to the field resistance and the load torque is increased by 50%.

Armature resistance = 0.15 \( \Omega \), field resistance = 0.1 \( \Omega \).

It may be assumed that flux per pole is proportional to the field.

Solution. In a series motor, prior to magnetic saturation

\[ T \propto \Phi I_a \propto I_a^2 \]; \( T_1 \propto I_a^2 \propto 40^2 \) ... (i)

If \( I_{a2} \) is the armature current (or motor current) in the second case when divertor is used, then only \( I_{a2}/2 \) passes through the series field winding.

\[ \Phi_2 \propto I_{a2}/2 \text{ and } T_2 \propto \Phi_2 I_{a2} \propto (I_{a2}/2) \times I_{a2} \propto I_{a2}^2/2 \] ... (ii)

From (i) and (ii), we get

\[ \frac{T_2}{T_1} = \frac{I_{a2}^2}{2 \times 40^2} \]

Also

\[ T_2/T_1 = 1.5 \]; \( 1.5 = I_{a2}/2 \times 40^2 \)

\[ I_{a2} = \sqrt{1.5 \times 2 \times 40^2} = 69.3 \, A \); \( \Phi_2 = \Phi_1 \times \frac{I_{a2}}{I_{a1}} = 220 \, V \); \( N_1 = 700 \, \text{r.p.m.} \); \( N_2 = ? \)

\[ \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} = \frac{\Phi_2}{\Phi_1} \]; \( \frac{N_2}{700} = \frac{206.14}{210} \times \frac{40}{69.3/2} \); \( N_2 = 794 \, \text{r.p.m.} \)

Example 30.38. A 4-pole, 250 V d.c. series motor takes 20 A and runs at 900 r.p.m. Each field coil has resistance of 0.025 ohm and the resistance of the armature is 0.1 ohm. At what speed will the motor run developing the same torque if:

(i) a divertor of 0.2 ohm is connected in parallel with the series field

(ii) rearranging the field coils in two series and parallel groups

Assume unsaturated magnetic operation.

(Electric Drives and their Control, Nagpur Univ. 1993)

Solution. The motor with its field coils all connected in series is shown in Fig. 30.15 (a). Here, \( N_1 = 900 \, \text{r.p.m.} \), \( E_{b1} = 250 - 20 \times (0.1 + 4 \times 0.025) = 246 \, V \).

In Fig. 30.15 (b), a divertor of resistance 0.2 \( \Omega \) has been connected in parallel with the series field coils. Let \( I_{a2} \) be the current drawn by the motor under this condition. As per current-divider rule, part of the current passing through the series fields is \( I_{a2} \times 0.2/(0.1 + 0.2) = 2 I_{a2}/3 \). Obviously, \( \Phi_2 \propto 2 I_{a2}/3 \).

* The combined resistance of series field winding and the divertor is 0.1/2 = 0.05 \( \Omega \). Hence, the total resistance = 0.15 + 0.05 = 0.2 \( \Omega \), in example 30.37.
Now, \( T_1 \propto \Phi_1 I_{a1} \propto I_{a1}^2 \); \( T_2 \propto \Phi_2 I_{a2} \propto (2 I_{a2}/3) I_{a2} \propto 2 I_{a2}^2/3 \)

Since 
\[
T_1 = T_2; \quad \therefore I_{a1} = 2 I_{a2}^2/3 \quad \text{or} \quad 20^2 = 2 I_{a2}^2/3; \quad \therefore I_{a2} = 24.5 \text{ A.}
\]

Combined resistance of the field and divertor is \( 0.2 \times 0.1/0.3 = 0.667 \) \( \Omega \); Arm. circuit resistance
\[
= 0.1 + 0.0667 = 0.1667 \quad \Omega; \quad E_{b2} = 250 - 24.5 \times 0.1667 = 250 - 4.1 = 245.9 \text{ V}
\]

\[
\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}; \quad \frac{N_2}{900} = \frac{245.9}{246} \times \frac{20}{(2/3) 24.5}; \quad N_2 = 1102 \text{ r.p.m.} \quad \text{...}(\because \Phi_2 \propto 2 I_{a2}/3)
\]

![Fig. 30.15](image)

(ii) In Fig. 30.15 (c), the series field coils have been arranged in two parallel groups. If the motor current is \( I_{a2} \), then it is divided equally between the two parallel paths. Hence, \( \Phi_2 \propto I_{a2}/2 \).

Since torque remains the same,
\[
T_1 \propto \Phi_1 I_{a1} \propto I_{a1}^2 \propto 20^2; \quad T_2 \propto \Phi_2 I_{a2} \propto (I_{a2}/2) I_{a2} \propto I_{a2}^2/2
\]

Since
\[
T_1 = T_2; \quad \therefore 20^2 = I_{a2}^2/2; \quad I_{a2} = 28.28 \text{ A}
\]

Combined resistance of the two parallel paths = \( 0.05/2 = 0.025 \) \( \Omega \)

Total arm. circuit resistance
\[
= 0.1 + 0.025 = 0.125 \text{ \( \Omega \)}
\]

\[
\therefore E_{b2} = 250 - 28.28 \times 0.125 = 246.5 \text{ V}
\]

\[
\frac{N_2}{900} = \frac{246.5}{246} \times \frac{20}{28.28/2}; \quad N_2 = 1275 \text{ r.p.m.}
\]

**Example 30.39.** A 4-pole, 230 V series motor runs at 1000 r.p.m., when the load current is 12 A. The series field resistance is 0.8 \( \Omega \) and the armature resistance is 1.0 \( \Omega \). The series field coils are now regrouped from all in series to two in series with two parallel paths. The line current is now 20 A. If the corresponding weakening of field is 15%, calculate the speed of the motor.

(Electrotechnology-I, Gwahati Univ. 1987)
Solution. \[
\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}, \quad E_{b1} = 230 - 12 \times 1.8 = 208.4 \text{ V}, \text{ as in Fig. 30.16(a)}
\]

For circuit in Fig. 30.16(b),

\[
E_{b2} = 230 - 20(1 + 0.4/2) = 206 \text{ V} ; \\
\Phi_2 = 0.85 \Phi_1 \text{ or } \Phi_1/\Phi_2 = 1/0.85
\]

\[
\therefore \quad \frac{N_2}{1000} = \frac{206}{208.4 \times 0.85} \Rightarrow N_2 = 1163 \text{ r.p.m.}
\]

Example 30.40. A 200 V, d.c. series motor runs at 500 r.p.m. when taking a line current of 25 A. The resistance of the armature is 0.2 \( \Omega \) and that of the series field 0.6 \( \Omega \). At what speed will it run when developing the same torque when armature diverton of 10 \( \Omega \) is used? Assume a straight line magnetisation curve.

(D.C. Machines, Jadavpur Univ. 1988)

Solution. Resistance of motor = 0.2 + 0.6 = 0.8 \( \Omega \)

\[
E_{b1} = 200 - (25 \times 0.8) = 180 \text{ V}
\]

Let the motor input current be \( I_2 \), when armature diverton is used, as shown in Fig. 30.17.

Series field voltage drop = 0.6 \( I_2 \)

\[
\therefore \quad \text{P.D. at brushes} = 200 - 0.6 I_2
\]

\[
\therefore \quad \text{Arm. divertor current} = \left( \frac{200 - 0.6 I_2}{10} \right) \text{ A}
\]

\[
\therefore \quad \text{Armature current} = I_2 \left( \frac{200 - 0.6 I_2}{10} \right)
\]

\[
\therefore \quad I_{a2} = \frac{10.6 I_2 - 200}{10}
\]

As torque in both cases is the same,

\[
\therefore \Phi_1 I_{a1} = \Phi_2 I_{a2}
\]

\[
\therefore \quad 25 \times 25 = I_2 \left( \frac{10.6 I_2 - 200}{10} \right) \text{ or } 6250 = 10.6 I_2^2 - 200 I_2
\]

or \( 0.6 I_2^2 - 200 I_2 - 6250 = 0 \) or \( I_2 = 35.6 \) A

P.D. at brushes in this case = 200 - (35.6 \times 0.6) = 178.6 V

\[
\therefore \quad I_{a2} = \frac{10.6 \times 35.6 - 200}{10} = 17.74 \text{ A};
\]

\[
E_{b2} = 178.6 - (17.74 \times 0.2) = 175 \text{ V}
\]

\[
\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \text{ or } \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_1}{I_2}
\]

\[
\therefore \quad \frac{N_2}{500} = \frac{175 \times 25}{180 \times 35.6} \Rightarrow N_2 = 314 \text{ r.p.m.}
\]

Example 30.41. A series motor is running on a 440 V circuit with a regulating resistance of \( R \) \( \Omega \) for speed adjustment. The armature and field coils have a total resistance of 0.3 \( \Omega \). On a certain load with \( R = 0 \), the current is 20 A and speed is 1200 r.p.m. With another load and \( R = 3 \) \( \Omega \), the current is 15 A. Find the new speed and also the ratio of the two values of the power outputs of the motor. Assume the field strength at 15 A to be 80\% of that at 20 A.

Solution.

\[
I_{a1} = 20 \text{ A}, \quad R_a = 0.3 \text{ } \Omega; \quad E_{b1} = 440 - (20 \times 0.3) = 434 \text{ V}
\]

\[
I_{a2} = 15 \text{ A}, \quad R_a = 3 + 0.3 = 3.3 \text{ } \Omega; \quad E_{b2} = 440 - (3.3 \times 15) = 390.5 \text{ V}
\]

\[
\Phi_2 = 0.8 \Phi_1, \quad N_1 = 1200 \text{ r.p.m.}
\]
Using \( \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2} \), we get \( N_2 = 1200 \times \frac{390.5}{434} \times \frac{1}{0.8} = 1350 \text{ r.p.m.} \)

Now, in a series motor,

\[ T \propto \Phi I_a \text{ and power } P \propto T \times N \text{ or } P \propto \Phi N I_a \]

\[ \therefore \quad P_1 \propto \Phi_1 \times 1200 \times 20 \quad \text{and} \quad P_2 \propto 0.8 \Phi_1 \times 1350 \times 15 \]

\[ \therefore \quad \frac{P_1}{P_2} = \frac{1200 \times 20 \Phi_1}{1350 \times 15 \times 0.8 \Phi_1} = 1.48 \]

Hence, power in the first case is 1.48 times the power in the second case.

**Example 30.42.** A d.c. series with an unsaturated field and negligible resistance, when running at a certain speed on a given load takes 50 A at 460 V. If the load-torque varies as the cube of the speed, calculate the resistance required to reduce the speed by 25%.

*(Nagpur, Univ. November 1999, Madras Univ. 1987)*

**Solution.** Let the speed be \( \omega \) radians/sec and the torque, \( T_{em} \) Nw-m developed by the motor.

Hence power handled = \( T. \omega \) watts

Let load torque be \( T_L \),

\[ T_L \propto \omega^3 \]

\[ T_L = T_{em}, \quad T_{em} \propto (50)^2 \]

Hence Load power = \( T_L \omega \)

Since no losses have to be taken into account, \( 50^2 \propto \omega^3 \)

Armature power, \( 460 \times 50 \propto \omega^4 \)

Based on back e.m.f. relationship, \( E_b \propto \omega I_a \)

\[ 460 \propto \omega \times 50 \]

To reduce the speed by 25%, operating speed = 0.75 \( \omega \) rad/sec

Let the new current be \( I \).

From Load side torque \( \propto (0.75 \omega)^3 \)

From electro-mech side, \( T \propto I^2 \)

\[ I^2 \propto (0.75 \omega)^3 \]

Comparing similar relationship in previous case,

\[ \frac{I^2}{50^2} = \frac{(0.75 \omega)^3}{\omega^3} = 0.75^3 \]

\[ I^2 = 50^2 \times 0.422 = 1055 \]

\[ I = 32.48 \text{ amp} \]

\[ E_{b2} \propto I \times \text{speed} \]

\[ \propto I \times 0.75 \omega \]

\[ \propto 32.48 \times 0.75 \omega \]

\[ E_{b2} = \frac{32.48 \times 0.75}{460} \]

\[ E_{b2} = 224 \text{ volts} \]

If \( R \) is the resistance externally connected in series with the motor.

\[ E_{b2} = 460 - 32.48 \times R = 224 \]

\[ \therefore \quad R = 7.266 \text{ ohms} \]

Previous armature power = \( 460 \times 50 \times 10^{-3} = 23 \text{ kW} \)
New armature power should be
\[ 23 \times (0.75)^4 = 7.28 \text{ kW} \]
With \( E_b \) as 224 V and current as 32.48 amp
\[ \text{Armature power} = 224 \times 32.48 \text{ watts} \]
\[ = 7.27 \text{ kW} \]

Thus, the final answer is checked by this step, since the results agree.

**Example 30.43.** A d.c. series motor drives a load, the torque of which varies as the square of speed. The motor takes a current of 15 A when the speed is 600 r.p.m. Calculate the speed and current when the motor-field-winding is shunted by a divertor of equal resistance as that of the field winding. Neglect all motor losses and assume the magnetic circuit as unsaturated.

**(Bharathithasan Univ. April 1997)**

**Solution.** Let the equations governing the characteristics of series motor be expressed as follows, with no losses and with magnetic circuit unsaturated.

Torque developed by motor = Load torque
\[ k_m \times i_{se} \times I_a = k_L \times (600)^2 \]
where \( k_m, k_L \) are constants
\[ I_a = \text{Armature current} \]
\[ I_{se} = \text{series field current} \]

In the first case
\[ i_{se} = I_a \]
With divertor,
\[ i_{se} = i_d = 0.5 I_a \]
Since, the resistances of divertor and series field are equal.

In the first case, \[ k_m \times 15 \times 15 = k_L \times (600)^2 \]
Let the supply voltage be \( V_L \) volts.

Since Losses are to be neglected, armature receives a power of \( (V_L \times 15) \) watts.

Case (ii) Let the new speed = \( N_2 \) r.p.m. and the new armature current = \( I_{a2} \) amp

So that new series-field current = 0.5 \( I_{a2} \)

Torque developed by motor = Load torque
\[ k_m \times (0.5 I_{a2}) \times I_{a2} = k_L \times (N_2)^2 \]

From equations (a) and (b) above,
\[ \frac{0.5 I_{a2}^2}{225} = \frac{N_2^2}{600^2} \]
\[ \frac{N_2^2}{I_{a2}^2} = \frac{600^2}{450} = 800 \]

or
\[ N_2 I_{a2} = 28.28 \]

Now armature receives a power of \( I_{a2} V_L \) watts. Mechanical outputs in the two cases have to be related with these electrical-power-terms.
\[ k_L = (600)^2 \times 2600/60 = 15 V_L \]
\[ k_L = N^2 \times 2 N/60 = I_{a2} V_L \]

From these two equations,
\[ N^3/600^3 = I_{a2}/15 \]
From (c) and (g),
\[ N_2 I_{a2} = 18,000 \]
From (d) and (h), \( N_2 = 713.5 \text{ r.p.m.} \)
And \( l_{a2} = 25.23 \text{ amp} \)

**Additional Correlation:** Since the load-torque is proportional to the square of the speed, the mechanical output power is proportional to the cube of the speed. Since losses are ignored, electrical power (input) must satisfy this proportion.

\[
\left( \frac{15 V_L}{25.23 V_L} \right) = \left( \frac{600}{713.5} \right)^3
\]

L.H.S. = 0.5945, R.H.S. = 0.5947

Hence, correlated and checked.

### 30.4. Merits and Demerits of Rheostatic Control Method

1. Speed changes with every change in load, because speed variations depend not only on controlling resistance but on load current also. This double dependence makes it impossible to keep the speed sensibly constant on rapidly changing loads.

2. A large amount of power is wasted in the controller resistance. Loss of power is directly proportional to the reduction in speed. Hence, efficiency is decreased.

3. Maximum power developed is diminished in the same ratio as speed.

4. It needs expensive arrangement for dissipation of heat produced in the controller resistance.

5. It gives speeds below the normal, not above it because armature voltage can be decreased (not increased) by the controller resistance.

This method, therefore, employed when low speeds are required for a short period only and that too occasionally as in printing machines and for cranes and hoists where motor is continually started and stopped.

**Advantages of Field Control Method**

This method is economical, more efficient and convenient though it can give speeds above (not below) the normal speed. The only limitation of this method is that commutation becomes unsatisfactory, because the effect of armature reaction is greater on a weaker field.

It should, however, be noted that by combining the two methods, speeds above and below the normal may be obtained.

### 30.5. Series-parallel Control

In this system of speed control, which is widely used in electric traction, two or more similar mechanically-coupled series motors are employed. At low speeds, the motors are joined in series Fig. 30.19 (a) and for high speeds, are joined in parallel Fig. 30.19 (b).

When in series, the two motors have the same current passing through them, although the voltage across each motor is \( V/2 \) i.e., half the supply voltage. When joined in parallel, voltage across each machines is \( V \), though current drawn by each motor is \( I/2 \).

**When in Parallel**

Now speed \( \propto E_b / \phi \propto E_b / \text{current} \)
(being series motors)
Since $E_b$ is approximately equal to the applied voltage $V$:

\[
\therefore \quad \text{speed} \propto \frac{V}{I/2} \propto \frac{2V}{I}
\]

Also,

\[
\text{torque} \propto \Phi I \propto I^2
\]

\[
\therefore \quad T \propto (I/2)^2 \propto I^2/4 \quad \cdots \quad (i)
\]

**When in Series**

Here

\[
\text{speed} \propto \frac{E_b}{\Phi} \propto \frac{V/2}{I} \propto \frac{V}{2I} \quad \cdots \quad (iii)
\]

This speed is one-fourth of the speed of the motors when in parallel.

Similarly

\[
T \propto \Phi I \propto I^2
\]

The torque is four times that produced by motors when in parallel.

This system of speed control is usually combined with the variable resistance method of control described in Art. 30.3 (2).

The two motors are started up in series with each other and with variable resistance which is cut out in sections to increase the speed. When all the variable series resistance is cut out, the motors are connected in parallel and at the same time, the series resistance is reinserted. The resistance is again reduced gradually till full speed is attained by the motors. The switching sequence is shown in Fig. 30.20. As the variable series controller resistance is not continuously rated, it has to be cut out of the circuit fairly quickly although in the four running positions $A$, $B$, $C$ and $D$, it may be left in circuit for any length of time.

**Example 30.44.** Two series motors run at a speed of 500 r.p.m. and 550 r.p.m. respectively when taking 50 A at 500 V. The terminal resistance of each motor is 0.5 $\Omega$. Calculate the speed of the combination when connected in series and coupled mechanically. The combination is taking 50 A on 500 V supply.

(Electrical Machinery-I, Mysore Univ. 1985)

**Solution.** First Motor

\[
E_{b1} = 500 - (50 \times 0.5) = 475 \text{ V} \quad ; \quad I = 50 \text{ A}
\]

Now,

\[
N_1 = E_{b1}/\Phi_1 \quad \text{or} \quad E_{b1} = N_1 \Phi_1 \quad \text{or} \quad E_{b1} = k N_1 \Phi_1
\]

\[
475 = k \times 500 \times \Phi_1 \quad \therefore \quad k \Phi_1 = 475/500
\]

**Second Motor**

\[
E_{b2} = 500 - (50 \times 0.5) = 475 \text{ V} \quad \text{Similarly,} \quad k \Phi_2 = 475/550
\]

When both motors are in series

\[
E'_b = 500 - (50 \times 2 \times 0.5) = 450 \text{ V}
\]
Now,
\[ E_b' = E_{b1} + E_{b2} = k \Phi_1 N + k \Phi_2 N \]
where \( N \) is the common speed when joined in series.
\[ \therefore \quad 450 = \frac{475}{500} N + \frac{475}{550} N \quad \therefore \quad N = 248 \text{ r.p.m.} \]

**Example 30.45.** Two similar 20 h.p. (14.92 kW), 250 V, 1000 r.p.m. series motors are connected in series with each other across a 250 V supply. The two motors drive the same shaft through a reduction gearing 5 : 1 and 4 : 1 respectively. If the total load torque on the shaft is 882 N-m, calculate (i) the current taken from the supply main (ii) the speed of the shaft and (iii) the voltage across each motor. Neglect all losses and assume the magnetic circuits to be unsaturated.

(Elect. Machines, Punjab Univ., 1991)

**Solution.** (i) Rated current of each motor = 14,920/250 = 59.68 A  
Back e.m.f.  
\[ E_b = 250 \text{ V} \] 
(neglecting \( I_a R_a \) drop)  
Now, \[ E_b \propto N \Phi \quad \text{As} \quad \Phi \propto I \quad \therefore \quad E_b \propto NI \]
\[ 250 = k \times \left(\frac{1000}{60}\right) \times 59.68 \quad \therefore \quad k = 0.25 \]
Let \( N_{sh} \) be the speed of the shaft.
Speed of the first motor \( N_1 = 5 N_{sh} \); Speed of the second motor \( N_2 = 4 N_{sh} \).
Let \( I \) be the new current drawn by the series set, then
\[ E_b' = E_{b1} + E_{b2} = kI \times 5 N_1 + kI \times N_2 = kI \times 5 N_{sh} + kI \times 4 N_{sh} \]
\[ 250 = 9 \times kI \times N_{sh} \quad \ldots (i) \]
Now,  
\[ \text{torque} \quad T = 0.159 \frac{E_b I}{N} = 0.159 \times \frac{kI N_{sh} \times I}{N_{sh}} = 0.159 k I^2 \]
Shaft torque due to gears of 1st motor = \( 5 \times 0.159 k I^2 \)
Shaft torque due to gears of 2nd motor = \( 4 \times 0.159 k I^2 \)
\[ \therefore \quad 882 = kI^2 (5 \times 0.159 + 4 \times 0.159) = 1.431 kI^2 \]
\[ \therefore \quad I^2 = \frac{882}{1.431 \times 0.25} = 2.449 \text{ A} \quad \therefore \quad I = 49.5 \text{ A} \]

(ii) From equation (i), we get  
\[ 250 = 9 \times 0.25 \times 49.5 \times N_{sh} \quad \therefore \quad N_{sh} = 2.233 \text{ r.p.m.} = 134 \text{ r.p.m.} \]

(iii) Voltage across the armature of 1st motor is  
\[ E_{b1}' = 5 kI N_{sh} = 5 \times 0.25 \times 49.5 \times 2.233 = 139 \text{ V} \]
Voltage across the armature of 2nd motor  
\[ E_{b2} = 4 kI N_{sh} = 4 \times 0.25 \times 49.5 \times 2.233 = 111 \text{ V} \]
Note that \( E_{b1}' \) and \( E_{b2} \) are respectively equal to the applied voltage across each motor because \( I_a R_a \) drops are negligible.

**30.6. Electric Braking**

A motor and its load may be brought to rest quickly by using either (i) Friction Braking or (ii) Electric Braking. The commonly-used mechanical brake has one drawback: it is difficult to achieve a smooth stop because it depends on the condition of the braking surface as well as on the skill of the operator.

The excellent electric braking methods are available which eliminate the need of brake lining levers and other mechanical gadgets. Electric braking, both for shunt and series motors, is of the following three types: (i) rheostatic or dynamic braking
(ii) plugging i.e., reversal of torque so that armature tends to rotate in the opposite direction and
(iii) regenerative braking.

Obviously, friction brake is necessary for holding the motor even after it has been brought to rest.

30.7. Electric Braking of Shunt Motors

(a) Rheostatic or Dynamic Braking

In this method, the armature of the shunt motor is disconnected from the supply and is connected across a variable resistance \( R \) as shown in Fig. 30.21 (b). The field winding is, however, left connected across the supply undisturbed. The braking effect is controlled by varying the series resistance \( R \). Obviously, this method makes use of generator action in a motor to bring it to rest.* As seen from Fig. 30.21 (b), armature current is given by

\[
I_a = \frac{E_b}{R + R_a} = \frac{\Phi Z N (P / A)}{R + R_a} = \frac{k_1 \Phi N}{R + R_a}
\]

Braking torque is given by

\[
T_B = \frac{1}{2 \pi} \Phi Z I_a \left( \frac{P}{A} \right)_N \text{N-m}
\]

\[
= \frac{1}{2 \pi} \Phi Z \left( \frac{P}{A} \right) \cdot \Phi Z N (P / A) = \frac{1}{2 \pi} \left( \frac{Z P}{A} \right)^2 \frac{\Phi^2 N}{R + R_a} = k_2 \Phi^2 N \therefore T_B \propto N
\]

Obviously, \( T_B \) decreases as motor slows down and disappear altogether when it comes to a stop.

(b) Plugging or Reverse Current Braking

This method is commonly used in controlling elevators, rolling mills, printing presses and machine tools etc.

In this method, connections to the armature terminals are reversed so that motor tends to run in the opposite direction (Fig. 30.22). Due to the reversal of armature connections, applied voltage \( V \) and \( E_b \) start acting in the same direction around the circuit. In order to limit the armature current to a reasonable value, it is necessary to insert a resistor in the circuit while reversing armature connections.

\[
I_a = \frac{V + E_b}{R + R_a} = \frac{V}{R + R_a} + \frac{E_b}{R + R_a}
\]

\[
= \frac{V}{R + R_a} + \frac{\Phi Z N (P / A)}{R + R_a} = \frac{V}{R + R_a} + k_1 \Phi Z
\]

\[
T_B = \frac{1}{2 \pi} \cdot \Phi Z I_a \left( \frac{P}{A} \right) = \frac{1}{2 \pi} \left( \frac{Z P}{A} \right) \cdot I_a = \frac{1}{2 \pi} \left( \frac{Z P}{A} \right) \left[ \frac{V}{R + R_a} + \frac{\Phi Z N (P / A)}{R + R_a} \right]
\]

* The motor while acting as a generator feeds current to the resistor dissipating heat at the rate of \( f R \). The current \( I_a \) produced by dynamic braking flows in the opposite direction, thereby producing a counter torque that slows down the machine.
\[
\frac{V}{R + R_a} \cdot \Phi + \frac{1}{2\pi} \left( \frac{ZP}{A} \right)^2 \cdot \Phi^2 N = k_2 \Phi + k_3 \Phi^2 N
\]

or
\[
T_b = k_4 + k_5 N, \text{ where } k_4 = \frac{1}{2\pi} \left( \frac{ZP}{A} \right) \left( \frac{V}{R + R_a} \right)
\]

and
\[
k_5 = \frac{1}{2\pi} \left( \frac{ZP}{A} \right)^2 \cdot \frac{1}{(R + R_a)}.
\]

Plugging gives greater braking torque than rheostatic braking. Obviously, during plugging, power is drawn from the supply and is dissipated by \( R \) in the form of heat. It may be noted that even when motor is reaching zero speed, there is some braking torque \( T_B = k_4 \) (see Ex. 30.47).

**Regenerative Braking**

This method is used when the load on the motor has over-hauling characteristic as in the lowering of the cage of a hoist or the downgrade motion of an electric train. Regeneration takes place when \( E_b \) becomes greater than \( V \). This happens when the over-hauling load acts as a prime mover and so drives the machines as a generator. Consequently, direction of \( I_a \) and hence of armature torque is reversed and speed falls until \( E_b \) becomes lower than \( V \). It is obvious that during the slowing down of the motor, power is returned to the line which may be used for supplying another train on an upgrade, thereby relieving the powerhouse of part of its load (Fig. 30.23).

For protective purposes, it is necessary to have some type of mechanical brake in order to hold the load in the event of a power failure.

**Example 30.46.** A 220 V compensated shunt motor drives a 700 N-m torque load when running at 1200 r.p.m. The combined armature compensating winding and interpole resistance is 0.008 \( \Omega \) and shunt field resistance is 55 \( \Omega \). The motor efficiency is 90%. Calculate the value of the dynamic braking resistor that will be capable of 375 N-m torque at 1050 r.p.m. The windage and friction losses may be assumed to remain constant at both speeds.

**Solution.**

Motor output = \( \omega T = 2\pi \times NT = 2\pi (1200/60) \times 700 = 87,965 \text{ W} \)

Power drawn by the motor = \( 87,965/0.9 = 97,740 \text{ W} \)

Current drawn by the motor = \( 97,740/220 = 444 \text{ A} \).

\[
\begin{align*}
I_{sh} &= 220/55 = 4 \text{ A} ; \\
I_{al} &= 444 - 4 = 440 \text{ A} \\
E_{b1} &= 220 - 440 \times 0.008 = 216.5 \text{ V} \\
I_{a2} &= 440 \times \frac{375}{100} = 2650 \text{ A} \\
N_2 &= \frac{E_{b2}}{E_{b1}} \text{ or } \frac{1050}{216.5} = \frac{E_{b2}}{E_{b1}}; E_{b2} = 189.4 \text{ V}
\end{align*}
\]

With reference to Fig. 30.23, we have
\[
189.4 = 2650 \times (0.008 + R); \quad R = 0.794 \text{ \( \Omega \)}
\]
30.8. Electric Braking of Series Motor

The above-discussed three methods as applied to series motors are as follows:

(a) Rheostatic (or Dynamic) Braking

The motor is disconnected from the supply, the field connections are reversed and the motor is connected in series with a variable resistance $R$ as shown in Fig. 30.24. Obviously, now, the machine is running as a generator. The field connections are reversed to make sure that current through field winding flows in the same direction as before (i.e., from $M$ to $N$) in order to assist residual magnetism. In practice, the variable resistance employed for starting purpose is itself used for braking purposes. As in the case of shunt motors,

$$T_B = k_2 \Phi^2 N = k_3 I_{n2} N$$  \hspace{1cm} \text{... prior to saturation}

(b) Plugging or Reverse Current Braking

As in the case of shunt motors, in this case also the connections of the armature are reversed and a variable resistance $R$ is put in series with the armature as shown in Fig. 30.25. As found in Art. 30.7 (b),

$$T_B = k_2 \Phi + k_2 \Phi^2 N$$

(c) Regenerative Braking

This type of braking of a series motor is not possible without modification because reversal of $I_a$ would also mean reversal of the field and hence of $E_b$. However, this method is sometimes used with traction motors, special arrangements being necessary for the purpose.

Example 30.47. A 400 V, 25 h.p. (18.65 kW), 45 r.p.m., d.c. shunt motor is braked by plugging when running on full load. Determine the braking resistance necessary if the maximum braking current is not to exceed twice the full-load current. Determine also the maximum braking torque and the braking torque when the motor is just reaching zero speed. The efficiency of the motor is 74.6% and the armature resistance is 0.2 $\Omega$.

(Electrical Technology, M.S. Univ. Baroda, 1988)

Solution.

F.L. Motor input current \[ I = \frac{18,650}{0.746} \times 400 = 62.5 \text{ A} \]

$I_a = 62.5 \text{ A}$ (neglecting $I_{sh}$);

$E_b = 400 - 62.5 \times 0.2 = 387.5 \text{ V}$

Total voltage around the circuit is \[ 400 + 387.5 = 787.5 \text{ V} \]

Max. braking current \[ 2 \times 62.5 = 125 \text{ A} \]

Total resistance required in the circuit \[ 787.5/125 = 6.3 \Omega \]

Braking resistance \[ R = 6.3 - 0.2 = 6.1 \Omega \]

Maximum braking torque will be produced initially when the motor speed is maximum i.e., 450 r.p.m. or 7.5 r.p.s.

Maximum value of $T_B = k_4 + k_5 N$ \hspace{1cm} \text{Art. 25.7(b)}

Now, \[ k_4 = \frac{1}{2\pi} \left( \frac{\Phi Z P}{A} \right) \left( \frac{V}{R + R_a} \right) \text{ and } k_5 = \frac{1}{2\pi} \left( \frac{\Phi Z P}{A} \right)^2 \left( \frac{1}{R + R_a} \right) \]
Now, \[ E_b = \Phi ZN (P/A) \text{; also } N = 450/60 = 7.5 \text{ r.p.s.} \]
\[ \therefore 387.5 = 7.5 (\Phi ZP/A) \text{ or } (\Phi ZP/A) = 51.66 \]
\[ \therefore k_4 = \frac{1}{2\pi} \times 51.66 \times \frac{400}{6.3} = 522 \text{ and } k_5 = \frac{1}{2\pi} \times (51.66)^2 \times \frac{1}{6.3} = 67.4 \]
\[ \therefore \text{Maximum } T_B = 522 + 67.4 \times 7.5 = 1028 \text{ N-m.} \]

30.9. Electronic Speed Control Method for DC Motors

Of late, solid-state circuits using semiconductor diodes and thyristors have become very popular for controlling the speed of a.c. and d.c. motors and are progressively replacing the traditional electric power control circuits based on thyratrons, ignitrons, mercury arc rectifiers, magnetic amplifiers and motor-generator sets etc. As compared to the electric and electromechanical systems of speed control, the electronic methods have higher accuracy, greater reliability, quick response and also higher efficiency as there are no \( I^2R \) losses and moving parts. Moreover, full 4-quadrant speed control is possible to meet precise high-speed standards.

All electronic circuits control the motor speed by adjusting either (i) the voltage applied to the motor armature or (ii) the field current or (iii) both.

DC motors can be run from d.c. supply if available or from a.c. supply after it has been converted into d.c. supply with the help of rectifiers which can be either half-wave or full-wave and either controlled (by varying the conduction angle of the thyristors used) or uncontrolled.

AC motors can be run on the a.c. supply or from d.c. supply after it has been converted into a.c. supply with the help of inverters (opposite of rectifiers).

As stated above, the average output voltage of a thyristor-controlled rectifier can be changed by changing its conduction angle and hence the armature voltage of the d.c. motor can be adjusted to control its speed.

When run on a d.c. supply, the armature d.c. voltage can be changed with the help of a thyristor chopper circuit which can be made to interrupt d.c. supply at different rates to give different average values of the d.c. voltage. If d.c. supply is not available, it can be obtained from the available a.c. supply with the help of uncontrolled rectifiers (using only diodes and not thyristors). The d.c. voltages so obtained can be then chopped with the help of a thyristor chopper circuit.

A brief description of rectifiers, inverters* and d.c. choppers would now be given before discussing the motor speed control circuits.

30.10. Uncontrolled Rectifiers

As stated earlier, rectifiers are used for a.c. to d.c. conversion i.e., when the supply is alternating but the motor to be controlled is a d.c. machine.

Fig. 30.26 (a) shows a half-wave uncontrolled rectifier. The diode \( D \) conducts only during positive half-cycles of the single-phase a.c. input i.e., when its anode \( A \) is positive with respect to its cathode \( K \). As shown, the average voltage available across the load (or motor) is 0.45 V where \( V \) is the r.m.s. value of the a.c. voltage (in fact, \( V = V_m / \sqrt{2} \)). As seen it is a pulsating d.c. voltage.

In Fig. 30.26 (b) a single-phase, full-wave bridge rectifier which uses four semiconductor diodes and provides double the voltage i.e., 0.9 V is shown. During positive input half-cycles when end \( A \) is positive with respect to end \( B \), diodes \( D_1 \) and \( D_2 \) conduct (i.e. opposite diodes) whereas during negative input half-cycles, \( D_2 \) and \( D_3 \) conduct. Hence, current flows through the load during both half-cycles in the same direction. As seen, the d.c. voltage supplied by a bridge rectifier is much less pulsating than the one supplied by the half-wave rectifier.

* Rectifiers convert a.c. power into d.c power, whereas inverters convert d.c. power into a.c. power. However, converter is a general term embracing both rectifiers and inverters.
30.11. Controlled Rectifiers

In these rectifiers, output load current (or voltage) can be varied by controlling the point in the input a.c. cycle at which the thyristor is turned ON with the application of a suitable low-power gate pulse. Once triggered (or fired) into conduction, the thyristor remains in the conducting state for the rest of the half-cycle *i.e.*, upto 180°. The firing angle \( \alpha \) can be adjusted with the help of a control circuit. When conducting, it offers no resistance *i.e.*, it acts like a short-circuit.

Fig. 30.27 (a) shows an elementary half-wave rectifier in which thyristor triggering is delayed by angle \( \alpha \) with the help of a phase-control circuit. As shown, the thyristor starts conducting at point A and not at point O because its gate pulse is applied after a delay of \( \alpha \). Obviously, the conduction angle is reduced from 180° to \((180° - \alpha)\) with a consequent decrease in output voltage whose value is given by

\[
V_L = \frac{V_m}{2\pi} (1 + \cos \alpha) = 0.16 \ V_m (1 + \cos \alpha) = 0.32 \ V_m \cos^2 \frac{\alpha}{2}
\]

where \( V_m \) is the peak value of a.c. input voltage. Obviously, \( V_L \) is maximum when \( \alpha = 0 \) and is zero when \( \alpha = 180° \).

Fig. 30.27 (b) shows the arrangement where a thyristor is used to control current through a load connected in series with the a.c. supply line.
The load current is given by

\[ I_L = \frac{V_L}{R_L} = \frac{V_m}{2\pi R_L} (1 + \cos \alpha) = \frac{V_m}{\pi R_L} \cos^2 \frac{\alpha}{2} \]

Fig. 30.28 (a) shows a single-phase, full-wave half-controlled rectifier. It is called half-controlled because it uses two thyristors and two diodes instead of four thyristors. During positive input half-cycle when A is positive, conduction takes place via \( T_1 \), load and \( D_1 \). During the negative half-cycle when B becomes positive, conduction route is via \( T_2 \), load and \( D_2 \).
The average output voltage \( V_L \) or \( V_{dc} \) is given by \( V_L = 2 \times \text{half-wave rectifier output} \)

\[
V_L = 2 \times \frac{V_m}{2\pi} (1 + \cos \alpha) = \frac{V_m}{\pi} (1 + \cos \alpha) = \frac{2V_m}{\pi} \cos^2 \frac{\alpha}{2}
\]

Similarly, Fig. 30.28 (b) shows a 4-diode bridge rectifier controlled by a single thyristor. The average load current through the series load is given by

\[
I_L = \frac{V_m}{\pi R_L} (1 + \cos \alpha) = \frac{2V_m}{\pi R_L} \cos^2 \frac{\alpha}{2}
\]

As seen from the figure, when \( A \) is positive, \( D_1 \) and \( D_3 \) conduct provided \( T \) has been fired. In the negative half-cycle, \( D_2 \) and \( D_4 \) conduct via the load.

### 30.12. Thyristor Choppers

Since thyristors can be switched ON and OFF very rapidly, they are used to interrupt a d.c. supply at a regular frequency in order to produce a lower (mean) d.c. voltage supply. In simple words, they can produce low-level d.c. voltage from a high-voltage d.c. supply as shown in Fig. 30.29.

The mean value of the output voltage is given by

\[
V_{dc} = V_L = V \frac{T_{ON}}{T_{ON} + T_{OFF}} = V \frac{T_{ON}}{T}
\]

![Fig. 30.29](image)

Fig. 30.30 (a) shows a simple thyristor chopper circuit along with extra commutating circuitry for switching \( T_1 \) OFF. As seen, \( T_1 \) is used for d.c. chopping, whereas \( R, T_2 \) and \( C \) are used for commutation purposes as explained below.

When \( T_1 \) is fired into conduction by its control circuit (not shown), current is set up through the load and commutation capacitor \( C \) gets charged via \( R \) with the polarity shown in the figure during this ON period.

For switching \( T_1 \) OFF, second thyristor \( T_2 \) is triggered into conduction allowing \( C \) to discharge through it (since it acts as a short-circuit while conducting) which reverse-biases \( T_1 \) thus turning it OFF. The discharge from \( C \) leaves \( T_2 \) with reverse polarity so that it is turned OFF, whereas \( T_1 \) is triggered into conduction again.

Depending upon the frequency of switching ON and OFF, the input d.c. voltage is cut into d.c. pulses as shown in Fig. 30.30 (c).

- For a fully-controlled bridge rectifier, its value is

\[
V_L = \frac{2V_m}{\pi} \cos \alpha
\]
In Fig. 30.30 (b), $T_1$ is the chopping thyristor, whereas $C$, $D$, $T_2$ and $L$ constitute the commutation circuitry for switching $T_1$ OFF and ON at regular intervals.

When $T_2$ is fired, $C$ becomes charged via the load with the polarity as shown. Next, when $T_1$ is fired, $C$ reverse-biases $T_2$ to OFF by discharging via $T_1$, $L$ and $D$ and then recharges with reverse polarity. $T_2$ is again fired and the charge on $C$ reverse-biases $T_1$ to non-conducting state.

It is seen that output (or load) voltage is present only when $T_1$ is ON and is absent during the interval it is OFF. The mean value of output d.c. voltage depends on the relative values of $T_{ON}$ and $T_{OFF}$. In fact, output d.c. voltage is given by

\[
V_{dc} = V_L \frac{T_{ON}}{T_{ON} + T_{OFF}} = V \frac{T_{ON}}{T}
\]

Obviously, by varying thyristor ON/OFF ratio, $V_L$ can be made any percentage of the input d.c. voltage $V$.

**Example 30.48.** The speed of a separately excited d.c. motor is controlled by a chopper. The supply voltage is 120 V, armature circuit resistance is 0.5 ohm, armature circuit inductance is 20 mH and motor constant is 0.05 V/r.p.m. The motor drives a constant load torque requiring an average current of 20 A. Assume motor current is continuous. Calculate (a) the range of speed control (b) the range of duty cycle. (Power Electronics-I, Punjab Univ. Nov. 1990)

**Solution.** The minimum speed is zero when $E_b = 0$

\[
V_t = E_b + I_a R_a = I_a \times R_a = 200 \times 0.5 = 10 \text{ V}
\]

Now,

\[
V_t = \frac{T_{ON}}{T} V = \alpha V, \therefore 10 = 120, \alpha = \frac{1}{12}
\]

Maximum speed corresponds to $\alpha = 1$ when $V_t = V = 120 \text{ V}$

\[
E_b = 120 - 20 \times 0.5 = 110 \text{ V}
\]

Now,

\[
N = \frac{E_b}{K_a} \Phi = 110/0.05 = 2200 \text{ r.p.m.}
\]

(a) Hence, speed range is from 0 to 2200 r.p.m.

(b) Range of duty cycle is from $\frac{1}{12}$ to 1.

### 30.13. Thyristor Inverters

Such inverters provide a very efficient and economical way of converting direct current (or voltage) into alternating current (or voltage). In this application, a thyristor serves as a controlled switch alternately opening and closing a d.c. circuit. Fig. 30.31 (a), shows a basic inverter circuit where an a.c. output is obtained by alternately opening and closing switches $S_1$ and $S_2$. When we
replace the mechanical switches by two thyristors (with their gate triggering circuits), we get the thyristor inverter in Fig. 30.31 (b).

![Fig. 30.31](image)

Before discussing the actual circuit, it is worthwhile to recall that thyristor is a latching device which means that once it starts conducting, gate loses control over it and cannot switch it OFF whatever the gate signal. A separate **commutating** circuitry is used to switch the thyristor OFF and thus enable it to perform ON-OFF switching function.

Suppose $T_1$ is fired while $T_2$ is still OFF. Immediately $I_1$ is set up which flows through $L$, one half of transformer primary and $T_1$. At the same time, $C$ is charged with the polarity as shown.

Next when $T_2$ is fired into condition, $I_2$ is set up and $C$ starts discharging through $T_1$ thereby reverse-biasing it to CUT-OFF.

When $T_1$ is again pulsed into condition, $I_1$ is set up and $C$ starts discharging thereby reverse-biasing $T_2$ to OFF and the process just described repeats. As shown in Fig. 30.31 (c), the output is an alternating voltage whose frequency depends on the switching frequency to thyristors $T_1$ and $T_2$.

### 30.14. Thyristor Speed Control of Separately-excited D.C. Motor

In Fig. 30.32, the bridge rectifier converts a voltage into d.c. voltage which is then applied to the armature of the separately-excited d.c. motor $M$.

As we know, speed of a motor is given by

$$N = \frac{V - I_a R_a}{\Phi} \left( \frac{A}{ZP} \right)$$

If $\Phi$ is kept constant and also if $I_a R_a$ is neglected, then, $N \propto V \propto$ voltage across the armature. The value of this voltage furnished by the rectifier can be changed by varying the firing angle $\alpha$ of the thyristor $T$ with the help of its control circuit. As $\alpha$ is increased i.e., thyristor firing is delayed
more, its conduction period is reduced and, hence, armature voltage is decreased which, in turn, decreases the motor speed. When $\alpha$ is decreased i.e., thyristor is fired earlier, conduction period is increased which increases the mean value of the voltage applied across the motor armature. Consequently, motor speed is increased. In short, as $\alpha$ increases, $V$ decreases and hence $N$ decreases. Conversely, as $\alpha$ decreases, $V$ increases and so, $N$ increases. The free-wheeling diode $D$ connected across the motor provides a circulating current path (shown dotted) for the energy stored in the inductance of the armature winding at the time $T$ turns OFF. Without $D$, current will flow through $T$ and bridge rectifier, prohibiting $T$ from turning OFF.

### 30.15. Thyristor Speed Control of a D.C. Series Motor

In the speed control circuit of Fig. 30.33, an $RC$ network is used to control the diac voltage that triggers the gate of a thyristor. As the a.c. supply is switched ON, thyristor $T$ remains OFF but the capacitor $C$ is charged through motor armature and $R$ towards the peak value of the applied a.c. voltage. The time it takes for the capacitor voltage $V_C$ to reach the breakover voltage of the diac depends on the setting of the variable resistor $T$. When $V_C$ becomes equal to the breakover voltage of diac, it conducts and a triggering pulse is applied to the thyristor gate $G$. Hence, $T$ is turned ON and allows current to pass through the motor. Increasing $R$ delays the rise of $V_C$ and hence the breakover of diac so that thyristor is fired later in each positive half cycle of the a.c. supply. It reduces the conduction angle of the thyristor which, consequently, delivers less power to the motor. Hence, motor speed is reduced.

If $R$ is reduced, time-constant of the $RC$ network is decreased which allows $V_C$ to rise to the breakover voltage of diac more quickly. Hence, it makes the thyristor fire early in each positive input half-cycle of the supply. Due to increase in the conduction angle of the thyristor, power delivered to the motor is increased with a subsequent increase in its speed. As before $D$ is the free-wheeling diode which provides circulating current path for the energy stored in the inductance of the armature winding.

### 30.16. Full-wave Speed Control of a Shunt Motor

Fig. 30.34 shows a circuit which provides a wide range of speed control for a fractional kW shunt d.c. motor. The circuit uses a bridge circuit for full-wave rectification of the a.c. supply. The shunt field winding is permanently connected across the d.c. output of the bridge circuit. The armature voltage is supplied through thyristor $T$. The magnitude of this

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* Like a triac, it has directional switching characteristics.
voltage (and hence, the motor speed) can be changed by turning $T_{ON}$ at different points in each half-cycle with the help of $R$. The thyristor turns OFF only at the end of each half-cycle. Free-wheeling diode $D_2$ provides a circulating current path (shows dotted) for the energy stored in the armature winding at the time $T$ turns OFF. Without $D_3$, This current would circulate through $T$ and the bridge rectifier thereby prohibiting $T$ from turning OFF.

At the beginning of each half-cycle, $T$ is the OFF state and $C$ starts charging up via motor armature, diode $D_2$ and speed-control variable resistor $R$ (it cannot charge through $R_1$ because of reverse-biased diode $D_1$). When voltage across $C$ i.e., $V_C$ builds up to the breakover voltage of diac, diac conducts and applies a sudden pulse to $T$ thereby turning it ON. Hence, power is supplied to the motor armature for the remainder of that half-cycle. At the end of each half-cycle, $C$ is discharged through $D_1$, $R_1$, and shunt field winding. The delay angle $\alpha$ depends on the time it takes $V_C$ to become equal to the breakover voltage of the diac. This time, in turn, depends on the time-constant of the $R$-$C$ circuit and the voltage available at point $A$. By changing $R$, $V_C$ can be made to build-up either slowly or quickly and thus change the angle $\alpha$ at will. In this way, the average value of the d.c. voltage across the motor armature can be controlled. It further helps to control the motor speed because it is directly proportional to the armature voltage.

Now, when load is increased, motor tends to slow down. Hence, $E_b$ is reduced. The voltage of point $A$ is increased because it is equal to the d.c. output voltage of the bridge rectifier minus back e.m.f. $E_b$. Since $V_A$ increases i.e., voltage across the $R$-$C$ charging circuit increases, it builds up $V_C$ more quickly thereby decreasing which leads to early switching ON of $T$ in each half-cycle. As a result, power supplied to the armature is increased which increases motor speed thereby compensating for the motor loading.

30.17. Thyristor Speed Control of a Shunt Motor

The speed of a shunt d.c. motor (upto 5 kW) may be regulated over a wide range with the help of the full-wave rectifier using only one main thyristor (or SCR) $T$ as shown in Fig. 30.35. The firing angle $\alpha$ of $T$ is adjusted by $R_1$ thereby controlling the motor speed. The thyristor and SUS (silicon unilateral switch) are reset (i.e., stop conduction) when each half-wave of voltage drops to zero. Before switching on the supply, $R_1$ is increased by turning it in the counter-clockwise direction. Next, when supply is switched ON, $C$ gets charged via motor armature and diode $D_1$ (being forward biassed). It means that it takes much longer for $V_C$ to reach the breakdown voltage of SUS due to large time constant of $R_1$-$C$ network. Once $V_C$ reaches that value, SUS conducts suddenly and triggers $T$ into conduction. Since thyristor starts conducting late (i.e., its $\alpha$ is large), it furnishes low voltage to start the motor. As speed selector $R_1$ is turned clockwise (for less resistance), $C$ charges up more rapidly (since time constant is decreased) to the breakover voltage of SUS thereby firing $T$ into conduction earlier. Hence, average value of the d.c. voltage across the motor armature increases thereby increasing its speed.

While the motor is running at the speed set by $R_1$, suppose that load on the motor is increased. In that case, motor will tend to slow down thereby decreasing armature back e.m.f. Hence, potential of point 3 will rise which will charge $C$ faster to the breakover voltage of SUS. Hence, thyristor will be fired earlier thereby applying greater armature voltage which will return the motor speed to its desired value. As seen, the speed is automatically regulated to offset changes in load.

The function of free-wheeling diode $D_2$ is to allow dissipation of energy stored in motor

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* It is a four-layer semiconductor diode with a gate terminal. Unlike diac, it conducts in one direction only.
armature during the time the full-wave rectified voltage drops to zero between half-cycles. If $D_2$ is not there, then decreasing armature current during those intervals would be forced to flow through $T$ thereby preventing its being reset. In that case, $T$ would not be ready to be fired in the next half-cycle.

Similarly, towards the end of each half cycle as points 1 and 5 decrease towards zero potential, the negative going gate $G$ turns $SUS$ on thereby allowing $C$ to discharge completely through $SUS$ and thyristor gate-cathode circuit so that it can get ready to be charged again in the next half-cycle.

### 30.18. Thyristor Speed Control of a Series D.C. Motor

Fig. 30.36 shows a simple circuit for regulating the speed of a d.c. motor by changing the average value of the voltage applied across the motor armature by changing the thyristor firing angle $\alpha$. The trigger circuit $R_1 - R_2$ can give a firing range of almost $180^\circ$. As the supply is switched on, full d.c. voltage is applied across $R_1 - R_2$. By changing the variable resistance $R_2$, drop across it can be made large enough to fire the $SCR$ at any desired angle from $0^\circ - 180^\circ$. In this way, output voltage of the bridge rectifier can be changed considerably, thus enabling a wide-range control of the motor speed. The speed control can be made somewhat smoother by joining a capacitor $C$ across $R_2$ as shown in the figure.

### 30.19. Necessity of a Starter

It has been shown in Art 29.3 that the current drawn by a motor armature is given by the relation

$$I_a = \frac{(V - E_b)}{R_a}$$

where $V$ is the supply voltage, $E_b$ the back e.m.f. and $R_a$ the armature resistance.

When the motor is at rest, there is, as yet, obviously no back e.m.f. developed in the armature. If, now, full supply voltage is applied across the stationary armature, it will draw a very large current because armature resistance is relatively small. Consider the case of a 440-V, 5 H.P. (3.73 kW) motor having a cold armature resistance of 0.25 $\Omega$ and a full-load current of 50 A. If this motor is started from the line directly, it will draw a starting current of $440/0.25 = 1760$ A which is $1760/50 = 35.2$ times its full-load current. This excessive current will blow out the fuses and, prior to that, it will damage the commutator and brushes etc. To avoid this happening, a resistance is introduced in series with the armature (for the duration of starting period only, say 5 to 10 seconds) which limits the starting current to a safe value. The starting resistance is gradually cut out as the motor gains speed and develops the back e.m.f. which then regulates its speed.

Very small motors may, however, be started from rest by connecting them directly to the supply lines. It does not result in any harm to the motor for the following reasons:

1. Such motors have a relatively higher armature resistance than large motors, hence their starting current is not so high.
2. Being small, they have low moment of inertia, hence they speed up quickly.
3. The momentary large starting current taken by them is not sufficient to produce a large disturbance in the voltage regulation of the supply lines.

In Fig. 30.37 the resistance $R$ used for starting a shunt motor is shown. It will be seen that the starting resistance $R$ is in series with the armature and not with the motor as a whole. The field winding is connected directly across the lines, hence shunt field current is independent of the
resistance $R$. If $R$ was introduced in the motor circuit, then $I_{sh}$ will be small at the start, hence starting torque $T_s$ would be small ($\therefore T_s \propto \Phi I_s$) and there would be experienced some difficulty in starting the motor. Such a simple starter is shown diagrammatically in Fig. 30.38.

30.20. Shunt Motor Starter

The face-plate box type starters used for starting shunt and compound motors of ordinary industrial capacity are of two kinds known as three-point and four-point starters respectively.

30.21. Three-point Starter

The internal wiring for such a starter is shown in Fig. 30.39 and it is seen that basically the connections are the same as in Fig. 30.37 except for the additional protective devices used here. The three terminals of the starting box are marked $A$, $B$ and $C$. One line is directly connected to one armature terminal and one field terminal which are tied together. The other line is connected to point $A$ which is further connected to the starting arm $L$, through the overcurrent (or overload) release $M$.

To start the motor, the main switch is first closed and then the starting arm is slowly moved to the right. As soon as the arm makes contact with stud No. 1, the field circuit is directly connected across the line and at the same time full starting resistance $R_s$ is placed in series with the armature. The starting current drawn by the armature $= V/(R_s + R_a)$ where $R_s$ is the starting
resistance. As the arm is further moved, the starting resistance is gradually cut out till, when the arm reaches the running position, the resistance is all cut out. The arm moves over the various studs against a strong spring which tends to restore it to OFF position. There is a soft iron piece $S$ attached to the arm which in the full ‘ON’ or running position is attracted and held by an electromagnet $E$ energised by the shunt current. It is variously known as ‘HOLD-ON’ coil, LOW VOLTAGE (or NO-VOLTAGE) release.

It will be seen that as the arm is moved from stud NO. 1 to the last stud, the field current has to travel back through that portion of the starting resistance that has been cut out of the armature circuit. This results in slight decrease of shunt current. But as the value of starting resistance is very small as compared to shunt field resistance, this slight decreases in $I_{sh}$ is negligible. This defect can, however, be remedied by using a brass arc which is connected to stud No. 1 (Fig. 30.40). The field circuit is completed through the starting resistance as it did in Fig. 30.39.

Now, we will discuss the action of the two protective devices shown in Fig. 30.39. The normal function of the HOLD-ON coil is to hold on the arm in the full running position when the motor is in normal operation. But, in the case of failure or disconnection of the supply or a break in the field circuit, it is de-energised, thereby releasing the arm which is pulled back by the spring to the OFF position. This prevents the stationary armature from being put across the lines again when the supply is restored after temporary shut down. This would have happened if the arm were left in the full ON position. One great advantage of connecting the HOLD-ON coil in series with the shunt field is that, should the field circuit become open, the starting arm immediately springs back to the OFF position thereby preventing the motor from running away.

The overcurrent release consists of an electromagnet connected in the supply line. If the motor becomes overloaded beyond a certain predetermined value, then $D$ is lifted and short circuits the electromagnet. Hence, the arm is released and returns to OFF position.

The form of overload protection described above is becoming obsolete, because it cannot be made either as accurate or as reliable as a separate well-designed circuit breaker with a suitable time element attachment. Many a times a separated magnetic contactor with an overload relay is also used.

Often the motors are protected by thermal overload relays in which a bimetallic strip is heated by the motor current at approximately the same rate at which the motor is itself heating up. Above a certain temperature, this relay trips and opens the line contactor, thereby isolating the motor from the supply.

If it is desired to control the speed of the motor in addition, then a field rheostat is connected in the filed circuit as indicated in Fig.30.39. The motor speed can be increased by weakening the flux ($N \propto I/\Phi$). Obviously, there is a limit to the speed increase obtained in this way, although speed ranges of three to four are possible. The connections of a starter and speed regulator with the motor are shown diagrammatically in Fig. 30.41. But there is one difficulty with such an arrangement for speed control. If too much resistance is ‘cut in’ by the field rheostat, then field current is reduced very much so that it is unable to create enough electromagnetic pull to overcome the spring tension. Hence,
the arm is pulled back to OFF position. It is this undesirable feature of a three-point starter which makes it unsuitable for use with variable-speed motors. This has resulted in widespread application of four-point starter discussed below.

30.22. Four-point Starter
Such a starter with its internal wiring is shown, connected to a long-shunt compound motor in Fig. 30.42. When compared to the three-point starter, it will be noticed that one important change has been made i.e., the HOLD-ON coil has been taken out of the shunt field circuit and has been connected directly across the line through a protecting resistance as shown. When the arm touches stud No. 1, then the line current divides into three parts (i) one part passes through starting resistance $R_s$, series field and motor armature (ii) the second part passes through the shunt field and its field rheostat $R_i$ and (iii) the third part passes through the HOLD-ON coil and current-protecting resistance $R$. It should be particularly noted that with this arrangement any change of current in the shunt field circuit does not at all affect the current passing through the HOLD-ON coil because the two circuits are independent of each other. It means that the electromagnetic pull exerted by the HOLD-ON coil will always be sufficient and will prevent the spring from restoring the starting arm to OFF position no matter how the field rheostat or regulator is adjusted.

### 30.23. Starter and Speed-control Rheostats

Sometimes, for convenience, the field rheostat is also contained within the starting box as shown in Fig. 30.43. In this case, two arms are used. There are two rows of studs, the lower ones being connected to the armature. The inside starting arm moves over the lower studs on the starting resistor, whereas the outside field lever moves over the upper ones on the field rheostat. Only the outside field arm is provided with an operating handle. While starting the motor, the two arms are moved together, but field lever is electrically inoperative because the field current flows directly from the starting arm through the brass arc to HOLD-ON coil and finally to the shunt field winding. At the end of the starting period, the starting arm is attracted and held in FULL-ON position by the HOLD-ON coil, and the contact between the starting arm and brass arc is broken thus forcing field current to pass through the field rheostat. The field lever can be moved back to increase the motor speed. It will be seen that now the upper row of contacts is operative because starting arm no longer touches the brass arc.

When motor is stopped by opening the main switch, the starting arm is released and on its way back it strikes the field lever so that both arms are returned simultaneously to OFF position.

### 30.24. Starting and Speed Control of Series Motor

For starting and speed control of series motor either a face-plate type or drum-type controller is used which usually has the reversing feature also. A face-plate type of reversing controller is shown in Fig. 30.44.

Except for a separate overload circuit, no inter-locking or automatic features are required because the operator watches the performance continuously.

As shown, the regulating lever consists of three pieces separated by strips of insulation. The outside parts form the electrical connections and the middle one is insulated from them. By moving the regulating lever, resistance can be cut in and out of the motor circuit. Reversing is obtained by moving the lever in the opposite direction as shown, because in that case, connections to the armature are reversed. Such an arrangement is employed where series motors are used as in the case of cranes, hoists and streetcars etc.
However, for adjustable speed service in connection with the operation of machine tools, a drum controller is preferred. It is called 'controller' because in addition to accelerating the motor to its normal speed, it provides the means for reversing the direction of the motor. Other desirable features such as safety protection against an open field or the temporary failure of power supply and overloads are frequently provided in this type of controller.

The controller consists of armature resistance grids of cross-section sufficient to carry the full-load operating current continuously and are used for adjusting the motor speed to values lower than the base speed obtained with no external resistance in the armature of field circuit. As the operating handle is gradually turned, the resistance is cut out of the armature circuit—there being as yet no resistance in the field cir-

![Diagram](image)

**Fig. 30.44**

...cuit at this stage, then when resistance in the armature circuit is completely cut out, further rotation of the handle inserts resistance into the field circuit. Turning of the handle in the opposite direction starts and speeds up the motor in the reverse direction.

### 30.25. Grading of Starting Resistance for Shunt Motors

$T_u$ would be small in designing shunt motor starters, it is usual to allow an overload of 50% for starting and to advance the starter a step when armature current has fallen to definite lower value. Either this lower current limit may be fixed or the number of starter steps may be fixed. In the former case, the number of steps are so chosen as to suit the upper and lower current limits whereas in the latter case, the lower current limit will depend on the number of steps specified. It can be shown that the resistances in the circuit on successive studs from geometrical progression, having a common ratio equal to lower current limit/upper current limit i.e., $I_2/I_1$.

In Fig. 30.45 the starter connected to a shunt motor is shown. For the sake of simplicity, four live studs have been taken. When arm A makes contact with stud No. 1, full shunt field is

![Diagram](image)

**Fig. 30.45**

...
established and at the same time the armature current immediately jumps to a maximum value $I_1$ given by $I_1 = V/R_1$ where $R_1 = \text{armature and starter resistance}$ (Fig. 30.45).

$I_1$ the maximum permissible armature current at the start ($I_{\text{max}}$) and is, as said above, usually limited to 1.5 times the full-load current of the motor. Hence, the motor develops 1.5 times its full-load torque and accelerates very rapidly. As the motor speeds up, its back e.m.f. grows and hence decreases the armature current as shown by curve $ab$ in Fig. 30.46.

When the armature current has fallen to some predetermined value $I_2$ (also called $I_{\text{min}}$) arm $A$ is moved to stud No. 2. Let the value of back e.m.f. be $E_{b1}$ at the time of leaving stud No. 1. Then

$$I_2 = \frac{V - E_{b1}}{R_1} \quad \ldots \text{(i)}$$

It should be carefully noted that $I_1$ and $I_2$ [(($I_{\text{max}}$) and ($I_{\text{min}}$)] are respectively the maximum and minimum currents of the motor. When arm $A$ touches stud No. 2, then due to diminution of circuit resistance, the current again jumps up to its previous value $I_1$. Since speed had no time to change, the back e.m.f. remains the same as initially.

$$I_1 = \frac{V - E_{b1}}{R_2} \quad \ldots \text{(ii)}$$

From (i) and (ii), we get $\frac{I_1}{I_2} = \frac{R_1}{R_2} \quad \ldots \text{(iii)}$

When arm $A$ is held on stud No. 2 for some time, then speed and hence the back e.m.f. increases to a value $E_{b2}$, thereby decreasing the current to previous value $I_2$, so that

$$I_2 = \frac{V - E_{b2}}{R_2} \quad \ldots \text{(iv)}$$

Similarly, on first making contact with stud No. 3, the current is

$$I_1 = \frac{V - E_{b2}}{R_3} \quad \ldots \text{(v)}$$

From (iv) and (v), we again get $\frac{I_1}{I_2} = \frac{R_2}{R_3} \quad \ldots \text{(vi)}$

When arm $A$ is held on stud No. 3 for some time, the speed and hence back e.m.f. increases to a new value $E_{b3}$, thereby decreasing the armature current to a value $I_2$ such that

$$I_2 = \frac{V - E_{b3}}{R_3} \quad \ldots \text{(vii)}$$

On making contact with stud No. 4, current jumps to $I_1$ given by

$$I_1 = \frac{V - E_{b3}}{R_4} \quad \ldots \text{(viii)}$$

From (vii) and (viii), we get $\frac{I_1}{I_2} = \frac{R_3}{R_4} \quad \ldots \text{(ix)}$

From (iii), (vi) and (ix), it is seen that

$$\frac{I_1}{I_2} = \frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} = K \quad \text{(say)} \quad \ldots \text{(x)}$$

Obviously,

$$R_1 = KR_4 \cdot R_2 = KR_3 \quad R_3 = K^2R_4 \quad R_1 = KR_2 = KKR_3 = K^3R_4$$

In general, if $n$ is the number of live studs and therefore $(n - 1)$ the number of sections in the starter resistance, then

$$R_1 = K^{n-1}R_4 \quad \text{or} \quad \frac{R_1}{R_4} = K^{n-1} \quad \text{or} \quad \left(\frac{I_1}{I_2}\right)^{n-1} = \frac{R_1}{R_4} \quad \ldots \text{from (x)}$$
Other variations of the above formula are

\[(a) \quad K^{n-1} = \frac{R_1}{R_a} = \frac{V}{I_1R_a} = \frac{V}{I_{\text{max}}R_a} \]

\[(b) \quad K^n = \frac{V}{I_1R_a} \cdot \frac{I_1}{I_2} = \frac{V}{I_2R_a} = \frac{V}{I_{\text{min}}R_a} \quad \text{and} \]

\[n = 1 + \frac{\log(V/R_aI_{\text{max}})}{\log K} \quad \text{from (a) above.} \]

Since \(R_1 = V/I_1\) and \(R_a\) are usually known and \(K\) is known from the given values of maximum and minimum currents (determined by the load against which motor has to start), the value of \(n\) can be found and hence the value of different starter sections.

**When Number of Sections is Specified.**

Since \(I_1\) would be given, \(R_1\) can be found from \(R_1 = V/I_1\).

Since \(n\) is known, \(K\) can be found from \(R_1/R_a = K^{n-1}\) and the lower current limit \(I_2\) from \(I_1/I_2 = K\).

**Example 30.49.** A 10 b.h.p. (7.46 kW) 200-V shunt motor has full-load efficiency of 85%. The armature has a resistance of 0.25 \(\Omega\). Calculate the value of the starting resistance necessary to limit the starting current to 1.5 times the full-load current at the moment of first switching on. The shunt current may be neglected. Find also the back e.m.f. of the motor, when the current has fallen to its full-load value, assuming that the whole of the starting resistance is still in circuit.

**Solution.**

Full-load motor current \(= 7.460/200 \times 0.85 = 43.88\) A

Starting current, \(I_1 = 1.5 \times 43.88 = 65.83\) A

\[R_1 = V/I_1 = 200/65.83 = 3.038\ \Omega ; \quad R_a = 0.25\ \Omega \]

\[\therefore \quad \text{Starting resistance} = R_1 - R_a = 3.038 - 0.25 = 2.788\ \Omega \]

Now, full-load current \(I_2 = 43.88\) A

Now,

\[I_2 = \frac{V - E_{bl}}{R_1} \]

\[\therefore \quad E_{bl} = V - I_2R_1 = 200 - (43.88 \times 3.038) = 67\ \text{V} \]

**Example 30.50.** A 220-V shunt motor has an armature resistance of 0.5 \(\Omega\). The armature current at starting must not exceed 40 A. If the number of sections is 6, calculate the values of the resistor steps to be used in this starter.

**Solution.** Since the number of starter sections is specified, we will use the relation.

\[R_1/R_a = K^{n-1} \quad \text{or} \quad R_1 = R_aK^{n-1} \]

Now,

\[R_1 = 220/40 = 5.5\ \Omega, \quad R_a = 0.4\ \Omega ; \quad n-1 = 6, \quad n = 7 \]

\[\therefore \quad 5.5 = 0.4 K^6 \quad \text{or} \quad K^6 = 5.5/0.4 = 13.75 \quad 6 \log_{10} K = \log_{10} 13.75 = 1.1383 ; \quad K = 1.548 \]

Now,

\[R_2 = R_1/K = 5.5/1.548 = 3.553\ \Omega \]

\[R_3 = R_2/K = 3.553/1.548 = 2.295\ \Omega \]

\[R_4 = 2.295/1.548 = 1.482\ \Omega \]

\[R_5 = 1.482/1.548 = 0.958\ \Omega \]

\[R_6 = 0.958/1.548 = 0.619\ \Omega \]

Resistance of 1st section \(= R_1 - R_2 = 5.5 - 3.553 = 1.947\ \Omega \)

" 2nd " = \(= R_2 - R_3 = 3.553 - 2.295 = 1.258\ \Omega \)

" 3rd " = \(= R_3 - R_4 = 2.295 - 1.482 = 0.813\ \Omega \)

" 4th " = \(= R_4 - R_5 = 1.482 - 0.958 = 0.524\ \Omega \)
5th " = $R_5 - R_6 = 0.958 - 0.619 = 0.339 \Omega$
6th " = $R_6 - R_a = 0.619 - 0.4 = 0.219 \Omega$

**Example 30.51.** Find the value of the step resistance in a 6-stud starter for a 5 h.p. (3.73 kW), 200-V shunt motor. The maximum current in the line is limited to twice the full-load value. The total Cu loss is 50% of the total loss. The normal field current is 0.6 A and the full-load efficiency is found to be 88%.

(D.C. Machines, Jadavpur Univ. 1988)

**Solution.**

Output $= 3,730 \text{ W}$
Total loss $= 4,238 - 3,730 = 508 \text{ W}$
Armature Cu loss alone $= 508/2 = 254 \text{ W}$
Input current $= 4,238/200 = 21.19 \text{ A}$
Armature current $= 21.19 - 0.6 = 20.59 \text{ A}$
$\therefore \ 20.59^2 R_a$ $= 254 \therefore \ R_a = 254/20.59^2 = 0.5989 \Omega$
Permissible input current $= 21.19 \times 2 = 42.38 \text{ A}$
Permissible armature current $= 42.38 - 0.6 = 41.78 \text{ A}$

$\therefore \ R_1 = 200/41.78 = 4.787 \Omega; \ n = 6 ; \ n - 1 = 5$

$\therefore \ 4.787 = K^5 \times 0.5989 \therefore K^5 = 4.787/0.5989 = 7.993$

or $5 \log K = \log 7.993 = 0.9027$

$\therefore \ \log K = 0.1805 \ ; \ K = 1.516$

Now

$R_2 = R_1/K = 4.789/1.516 = 3.159 \Omega$
$R_3 = 2.084 \Omega \ ; \ R_4 = 1.376 \Omega \ ; \ R_5 = 0.906 \Omega$

Resistance in 1st step $= R_1 - R_2 = 4.787 - 3.159 = 1.628 \Omega$
Resistance in 2nd step $= R_2 - R_3 = 3.159 - 2.084 = 1.075 \Omega$
Resistance in 3rd step $= R_3 - R_4 = 2.084 - 1.376 = 0.708 \Omega$
Resistance in 4th step $= R_4 - R_5 = 1.376 - 0.908 = 0.468 \Omega$
Resistance in 5th step $= R_5 - R_a = 0.908 - 0.5989 = 0.309 \Omega$

The various sections are shown in Fig. 28.47.

![Fig. 30.47](image)

**Example 30.52.** Design the resistance sections of a seven-stud starter for 36.775 kW, 400V, d.c. shunt motor. Full-load efficiency is 92%, total Cu losses are 5% of the input. Shunt field resistance is 200 Ω. The lower limit of the current through the armature is to be full-load value.

(Elec. Machines, Gujarat Univ. 1987)

**Solution.**

Output $= 36,775 \text{ W} \ ; \ \text{Input} = 36,775/0.92 = 39,980 \text{ W}$
Total Cu loss $= 0.05 \times 39,980 = 1,999W$
Shunt Cu loss $= V^2/R_{sh} = 400^2/200 = 800 \text{ W}$
Armature Cu loss = 1,999 - 800 = 1199 W
F.L. input current = 39,980/400 = 99.95 A
\[ I_{sh} = 400/200 = 2 A; \quad I_a = 99.95 - 2 = 97.95 A \]
\[ 97.95^2 R_a = 1199 \text{ W or } R_a = 0.125 \Omega \]

Now, minimum armature current equals full-load current i.e. \( I_a = 97.95 \text{ A} \). As seen from Art. 30.25 in its formula given in (b), we have

\[ K^n = \frac{V}{I_a R_a} \]

or

\[ K^2 = \frac{400/97.95 \times 0.125}{1.645} = 32.68 \]

\[ K = 32.68^{1/2} = 1.645 \]

\[ I_1 = \text{maximum permissible armature current} \]
\[ = K I_2 = 1.645 \times 97.94 = 161 \text{ A} \]

\[ R_1 = \frac{V}{I_1} = 400/161 = 2.483 \Omega \]
\[ R_2 = R_1/K = 2.483/1.645 = 1.51 \Omega \]
\[ R_3 = 1.51/1.645 = 0.917 \Omega \]
\[ R_4 = 0.917/1.645 = 0.557 \Omega \]
\[ R_5 = 0.557/1.645 = 0.339 \Omega \]

\[ R_6 = 0.339/1.645 = 0.206 \Omega \]
\[ R_7 = 0.206/1.645 = 0.125 \Omega \]

Resistance in 1st step = \( R_1 - R_2 = 0.973 \Omega \)
Resistance in 2nd step = \( R_2 - R_3 = 0.593 \Omega \)
Resistance in 3rd step = \( R_3 - R_4 = 0.36 \Omega \)
Resistance in 4th step = \( R_4 - R_5 = 0.218 \Omega \)
Resistance in 5th step = \( R_5 - R_6 = 0.133 \Omega \)
Resistance in 6th step = \( R_6 - R_a = 0.081 \Omega \)

The various starter sections are shown in Fig 30.48.

**Example 30.53.** Calculate the resistance steps for the starter of a 250-V d.c. shunt motor having an armature resistance of 0.125 \( \Omega \) and a full-load current of 150 A. The motor is to start against full-load and maximum current is not to exceed 200 A.

*(Elect. Engineering-I, Bombay Univ. 1989)*

**Solution.** As the motor is to start against its full-load, the minimum current is its F.L. current i.e. 150A. We will use the formula given in Art. 30.25.

\[ (I_1/I_2^{n-1}) = R_1/R_a \]

Here
\[ I_1 = 200 \text{ A; } I_2 = 150 \text{ A; } R_a = 0.125 \Omega ; \quad n = \text{No. of live studs} \]

\[ (200/150)^{n-1} = 1.25/0.125 = 10 \quad \text{or} \quad (4/3)^{n-1} = 10 \]

\[ (n-1) \log 4/3 = \log 10 \text{ or } (n-1) \times 0.1249 = 1 \]

\[ (n-1) = 1/0.1249 = 8 \]

Hence, there are 9 studs and 8 steps.

Now
\[ R_2 = R_1 \times I_2/I_1 = 1.25 \times 3/4 = 0.938 \Omega \]
\[ R_3 = 0.938 \times 3/4 = 0.703 \Omega \]
\[ R_4 = 0.703 \times 3/4 = 0.527 \Omega \]
\[ R_5 = 0.527 \times 3/4 = 0.395 \, \Omega \]
\[ R_6 = 0.395 \times 3/4 = 0.296 \, \Omega \]
\[ R_7 = 0.296 \times 3/4 = 0.222 \, \Omega \]
\[ R_8 = 0.222 \times 3/4 = 0.167 \, \Omega \]
\[ R_a = 0.167 \times 3/4 = 0.125 \, \Omega \]

\[ \therefore \text{Resistance of 1st element} = 1.25 - 0.938 = 0.312 \, \Omega \]

\[ \therefore 2\text{nd} \, \text{element} = 0.938 - 0.703 = 0.235 \, \Omega \]
\[ \therefore 3\text{rd} \, \text{element} = 0.703 - 0.527 = 0.176 \, \Omega \]
\[ \therefore 4\text{th} \, \text{element} = 0.527 - 0.395 = 0.132 \, \Omega \]
\[ \therefore 5\text{th} \, \text{element} = 0.395 - 0.296 = 0.099 \, \Omega \]
\[ \therefore 6\text{th} \, \text{element} = 0.296 - 0.222 = 0.074 \, \Omega \]
\[ \therefore 7\text{th} \, \text{element} = 0.222 - 0.167 = 0.055 \, \Omega \]
\[ \therefore 8\text{th} \, \text{element} = 0.167 - 0.125 = 0.042 \, \Omega \]

**Example 30.54.** The 4-pole, lap-wound armature winding of a 500-V, d.c. shunt motor is housed in a total number of 60 slots each slot containing 20 conductors. The armature resistance is 1.31 \( \Omega \). During the period of starting, the minimum torque is required to be 218 N-m and the maximum torque 1.5 times the minimum torque, find out how many sections the starter should have and calculate the resistances of these sections. Take the useful flux per pole to be 23 mWb.

*(Elect. Machinery-II, Bangalore Univ. 1991)*

**Solution.** From the given minimum torque, we will be able to find the minimum current required during starting. Now

\[ T_a = 0.159 \, \Phi \, ZI_a (P/A) \]
\[ 218 = 0.159 \times 23 \times 10^{-3} \times (60 \times 20) I_a \times (4/4) \quad \therefore I_a = 50 \, \text{A (approx.)} \]

Maximum current \( = 50 \times 1.5 = 75 \, \text{A} \)

\[ I_1 = 75 \, \text{A} \]; \( I_2 = 50 \, \text{A} \) \( \therefore I_1/I_2 = 75/50 = 1.5 \)

\[ R_4 = 500/75 = 6.667 \, \Omega \]

If \( n \) is the number of stater studs, then

\[ (I_1/I_2)^{n-1} = R_4/R_a \quad \text{or} \quad 1.5^{n-1} = 6.667/1.31 = 5.09 \]

\[ \therefore (n - 1) \log_{10} 1.5 = \log_{10} 5.09 \quad \therefore (n - 1) \times 0.1761 = 0.7067 \quad \therefore (n - 1) = 4 \text{ or } n = 5 \]

Hence, there are five studs and four sections.

\[ R_2 = R_1 \times I_2/I_1 = 6.667 \times 2/3 = 4.44 \, \Omega \]
\[ R_3 = 4.44 \times 2/3 = 2.96 \, \Omega \]; \( R_4 = 2.96 \times 2/3 = 1.98 \, \Omega \)

Resistance of 1st section \( = R_1 - R_2 = 6.67 - 4.44 = 2.23 \, \Omega \)

\[ \therefore 2\text{nd} \, \text{section} = R_2 - R_3 = 4.44 - 2.96 = 1.48 \, \Omega \]
\[ \therefore 3\text{rd} \, \text{section} = R_3 - R_4 = 2.96 - 1.98 = 0.98 \, \Omega \]
\[ \therefore 4\text{th} \, \text{section} = R_4 - R_a = 1.98 - 1.31 = 0.67 \, \Omega \]

### 30.26. Series Motor Starters

The basic principle employed in the design of a starter for series motor is the same as for a shunt motor *i.e.*, the motor current is not allowed to exceed a certain upper limit as the starter arm moves from one stud to another. However, there is one significant difference. In the case of a series motor, the flux does not remain constant but varies with the current because armature current is also the exciting current. The determination of the number of steps is rather complicated as illustrated in Example 30.55. It may however, be noted that the section resistances form a geometrical progression.
The face-plate type of starter formerly used for d.c. series motor has been almost entirely replaced by automatic starter in which the resistance steps are cut out automatically by means of a contactor operated by electromagnets. Such starters are well-suited for remote control.

However, for winch and crane motors where frequent starting, stopping, reversing and speed variations are necessary, drum type controllers are used. They are called controllers because they can be left in the circuit for any length of time. In addition to serving their normal function of starters, they also used as speed controllers.

**Example 30.55.** (a) Show that, in general, individual resistances between the studs for a rheostat starter for a series d.c. motor with constant ratio of maximum to minimum current at starting, are in geometrical progression, stating any assumptions made.

(b) Assuming that for a certain d.c. series motor the flux per pole is proportional to the starting current, calculate the resistance of each rheostat section in the case of a 50 b.h.p. (37.3 kW) 440-V motor with six sections.

The total armature and field voltage drop at full-load is 2% of the applied voltage, the full-load efficiency is 95% and the maximum starting current is 130% of full-load current.

**Solution.** (a) Let \( I_1 = \text{maximum current} \), \( I_2 = \text{minimum current} \)

\[ \Phi_1 = \text{flux/pole for } I_1; \quad \Phi_2 = \text{flux/pole for } I_2 \]

\[ \frac{I_1}{I_2} = K \quad \text{and} \quad \frac{\Phi_1}{\Phi_2} = \alpha. \]

Let us now consider the conditions when the starter arm is on the \( n \)th and \((n + 1)\)th stud. When the current is \( I_2 \), then \( E_b = V - I_2 R_n \).

If, now, the starter is moved up to the \((n + 1)\)th stud, then

\[ E_b' = \frac{\Phi_1}{\Phi_2} E_b = \alpha E_b \]

\[ R_{n+1} = \frac{V - E_b'}{I_1} = \frac{V - \alpha E_b}{I_1} = \frac{V - \alpha (V - I_2 R_n)}{I_1} = \frac{V}{I_1} (1 - \alpha) + \alpha \frac{I_2}{I_1} R_n \]

Now, \( V/I_1 = R_1 \)--the total resistance in the circuit when the starter arm is on the first stud.

\[ R_{n+1} = R_1 (1 - \alpha) + \frac{\alpha}{K} R_n \]

Similarly, by substituting \((n - 1)\) for \( n \), we get

\[ R_n = R_1 (1 - \alpha) + \frac{\alpha}{K} R_{n-1} \]

Therefore, the resistance between the \( n \)th and \((n + 1)\)th studs is

\[ r_n = R_n - R_{n+1} = \frac{\alpha}{K} R_{n-1} - \frac{\alpha}{K} R_n = \frac{\alpha}{K} (R_{n-1} - R_n) = \frac{\alpha}{K} r_{n-1} \]

\[ \frac{r_n}{r_{n-1}} = \frac{\alpha}{K} \frac{\Phi_1}{\Phi_2} \frac{I_2}{I_1} = b \quad \text{-- constant} \]

Obviously, the resistance elements form a geometrical progression series.

(b) Full-load input current \( = \frac{37,300}{440 \times 0.95} = 89.2 \text{ A} \)

Max. starting current \( I_1 = 1.3 \times 89.2 = 116 \text{ A} \)

Arm. and field voltage drop on full-load = 2% of 440 = 0.02 \times 440 = 8.8 \text{ V} \)

Resistance of motor = 8.8/89.2 = 0.0896 \text{ \Omega} \)

Total circuit resistance on starting, \( R_1 = V/I_1 = 440/116 = 3.79 \text{ \Omega} \)
Assuming straight line magnetisation, we have \( I_1 \propto \Phi_1 \) and \( I_2 \propto \Phi_2 \)

\[ \therefore \quad \frac{I_1}{I_2} = \frac{\Phi_1}{\Phi_2} \quad \therefore \quad \alpha = \frac{b}{\alpha K} = 1 \quad \therefore \quad r_n = b \times r_{n-1} \]

In other words, all sections have the same resistance.

\[ r = \frac{R_1 - R_{\text{motor}}}{\text{No. of sections}} = \frac{3.79 - 0.0896}{6} = 0.6176 \, \Omega \]

**Example 30.56.** A 75 h.p. (55.95 kW) 650-V, d.c. series tractions motor has a total resistance of 0.51 \( \Omega \). The starting current is to be allowed to fluctuate between 140 A and 100 A the flux at 140 A being 20% greater than at 100 A. Determine the number of steps required in the controller and the resistance of each step.

**Solution.** Let \( R_1 \) = total resistance on the first stud = 650/140 = 4.65 \( \Omega \)

When motor speeds up, then back e.m.f. is produced and current falls to \( I_2 \).

\[ V = E_{b1} + I_2 R_1 \quad \ldots (i) \]

When the starter moves to the next stud, the speed is still the same, but since current rises to \( I_1 \) for which flux is 1.2 times greater than for \( I_2 \), hence back e.m.f. becomes 1.2 \( E_{b1} \). Since resistance in the circuit is now \( R_2 \).

\[ V = I_1 R_2 + 1.2 E_{b1} \quad \ldots (ii) \]

From (i) and (ii) we get, \( 0.2 V = 1.2 I_2 R_1 - I_1 R_2 \)

\[ R_2 = (1.2 I_2/I_1) R_1 - 0.2 \frac{V}{I_1} = (1.2 I_2/I_1) R_1 - 0.2 \frac{E_{b1}}{I_1} = (1.2 I_2/I_1 - 0.2) R_1 \]

Similarly
\[ R_3 = (1.2 I_3/I_1) R_2 - 0.2 R_1 \]

In this way, we continue till we reach the value of resistance equal to the armature resistance. Hence, we obtain \( R_1, R_2, \ldots \) and also the number of steps.

In the present case, \( I_1 = 140 \, \text{A}, \quad I_2 = 100 \, \text{A}, \quad V = 650 \, \text{V} \) and \( I_1/I_2 = 100/140 = 1.14 \)

\[ R_1 = 4.65 \, \Omega; \quad R_2 = (1.2/1.4 - 0.2) 4.65 = 3.07 \, \Omega \]
\[ R_3 = (1.2/1.4) \times 3.07 - 0.2 \times 4.65 = 1.70 \, \Omega \]
\[ R_4 = (1.2/1.4) \times 1.7 - 0.2 \times 4.65 = 0.53 \, \Omega \]

We will stop here because \( R_4 \) is very near the value of the motor resistance. Hence, there are 4 studs and 3 sections or steps.

\[ R_1 - R_2 = 4.65 - 3.07 = 1.58 \, \Omega, \quad R_2 - R_3 = 3.07 - 1.7 = 1.37 \, \Omega \]
\[ R_3 - R_4 = 1.70 - 0.53 = 1.17 \, \Omega \]

**Note.** It will be seen that

\[ R_2 - R_3 = (1.2 I_2/I_1) (R_1 - R_2) \]
\[ R_3 - R_4 = (1.2 I_2/I_1) (R_2 - R_3) \]

and so on.

\[ \frac{R_2 - R_3}{R_1 - R_2} = \frac{R_3 - R_4}{R_2 - R_3} \]

or

\[ \frac{R_2 - R_3}{R_1 - R_2} = \frac{R_3 - R_4}{R_2 - R_3} = \frac{1.2}{1.14} = \frac{\Phi_1}{\Phi_2} = \frac{I_2}{I_1} = \frac{\Phi_1}{\Phi_2} \frac{I_2}{I_1} = \frac{\alpha}{b} \]

It is seen that individual resistances of various sections decrease in the ratio of \( \alpha/b \).

### 30.27. Thyristor Controller Starters

The moving parts and metal contacts etc., of the resistance starters discussed in Art. 30.21 can be eliminated by using thyristors which can short circuit the resistance sections one after another. A thyristor can be switched on to the conducting state by applying a suitable signal to its gate terminal. While conducting, it offers zero resistance in the forward (i.e., anode-to-cathode) direction and thus acts as a short-circuit for the starter resistance section across which it is connected. It can be switched off (i.e., brought back to the non-conducting state) by reversing the polarity of its anode-cathode voltage. A typical thyristor-controlled starter for d.c. motors is shown in Fig. 30.49.

After switching on the main supply, when switch \( S_1 \) is pressed, positive signal is applied to gate \( G \) of thyristor \( T_1 \) which is, therefore, turned ON. At the same time, shunt field gets established since it is directly connected across the d.c. supply. Consequently, motor armature current \( I_a \) flows via \( T_1 \),
$R_2$, $R_3$ and $R_4$ because $T_2$, $T_3$ and $T_4$ are, as yet in the non-conducting state. From now onwards, the starting procedure is automatic as detailed below:

1. As $S_1$ is closed, capacitor $C$ starts charging up with the polarity as shown when $I_a$ starts flowing.
2. The armature current and field flux together produce torque which accelerates the motor and load.

3. As motor speeds up, voltage provided by tachogenerator ($TG$) is proportionately increased because it is coupled to the motor.
4. At some motor speed, the voltage provided by $TG$ becomes large enough to breakdown Zener diode $Z_2$, and hence trigger $T_2$ into conduction. Consequently, $R_2$ is shorted out and now $I_a$ flows via motor armature, $T_1$, $T_2$, $R_3$ and $R_4$ and back to the negative supply terminal.
5. As $R_2$ is cut out, $I_a$ increases, armature torque increases, motor speed increases which further increases the voltage output of the tachogenerator. At some speed, $Z_3$ breaks down, thereby triggering $T_3$ into conduction which cuts out $R_3$.
6. After sometime, $R_4$ is cut out as $Z_4$ breaks down and triggers $T_4$ into conduction. In fact, Zener diodes $Z_2$, $Z_3$ and $Z_4$ can be rated for 1/3, 1/2 and 3/4 full speed respectively.

For stopping the motor, switch $S_2$ is closed which triggers $T_5$ into conduction, thereby establishing current flow via $R_1$. Consequently, capacitor $C$ starts discharging thereby reverse-biasing $T_1$ which stops conducting. Hence $I_a$ ceases and, at the same time, $T_2$, $T_3$ and $T_4$ also revert back to their non-conducting state.

Incidentally, it may be noted that the function of $C$ is to switch $T_1$, ON and OFF. Hence, it is usually called commutating capacitor.

The function of the diodes $D_1$ and $D_2$ is to allow the decay of inductive energy stored in the motor armature and field when supply is disconnected. Supply failure will cause the thyristors to block because of this current decay, thereby providing protection usually given by no-voltage release coil.

Recently, thyristor starting circuits have been introduced which use no starting resistance at all, thereby making the entire system quite efficient and optimized as regards starting time. These are based on the principle of ‘voltage chopping’ (Art. 30.12). By varying the chopping frequency, the ratio of the time the voltage is ON to the time it is OFF can be varied. By varying this ratio, the average voltage applied to the motor can be changed. A low average voltage is needed to limit the
armature current while the motor is being started and gradually the ratio is increased to reach the maximum at the rated speed of the motor.

**Tutorial Problems 32.2**

1. A shunt-wound motor runs at 600 r.p.m. from a 230-V supply when taking a line current of 50 A. Its armature and field resistances are 0.4 Ω and 104.5 Ω respectively. Neglecting the effects of armature reaction and allowing 2 V brush drop, calculate (a) the no-load speed if the no-load line current is 5A (b) the resistance to be placed in armature circuit in order to reduce the speed to 500 r.p.m. when motor is taking a line current of 50 A (c) the percentage reduction in the flux per pole in order that the speed may be 750 r.p.m. when the armature current is 30 A with no added resistance in the armature circuit. 
   
   [(a) 652 r.p.m. (b) 0.73 Ω (c) 1.73 %]  

2. The resistance of the armature of a 250-V shunt motor is 0.3 Ω and its full-load speed is 1000 r.p.m. Calculate the resistance to be inserted in series with the armature to reduce the speed with full-load torque to 800 r.p.m., the full-load armature current being 5A. If the load torque is then halved, at what speed will the motor run? Neglect armature reaction.  
   
   [0.94 Ω; 932 r.p.m.]  

3. A 230-V d.c. shunt motor takes an armature current of 20 A on a certain load. Resistance of the armature is 0.5 Ω. Find the resistance required in series with the armature to half the speed if (a) the load torque is constant (b) the load torque is proportional to the square of the speed.  
   
   [(a) 5.5 Ω (b) 23.5 Ω]  

4. A 230-V series motor runs at 1200 r.p.m. at a quarter full-load torque, taking a current of 16 A. Calculate its speed at half and full-load torques. The resistance of the armature brushes, and field coils is 0.25 Ω. Assume the flux per pole to be proportional to the current. Plot torque/speed graph between full and quarter-load.  
   
   [842 r.p.m.; 589 r.p.m.]  

5. A d.c. series motor drives a load the torque of which is proportional to the square of the speed. The motor current is 20 A when speed is 500 r.p.m. Calculate the speed and current when the motor field winding is shunted by a resistance of the same value as the field winding. Neglect all motor losses and assume that the magnetic field is unsaturated.  
   
   [595 r.p.m.; 33.64 A]  

   (Electrical Machines-I, Aligarh Muslim Univ. 1979)  

6. A d.c. series motor, with unsaturated magnetic circuit and with negligible resistance, when running at a certain speed on a given load takes 50 A at 500 V. If the load torque varies as the cube of the speed, find the resistance which should be connected in series with machine to reduce the speed by 25 per cent.  
   
   [7.89 Ω]  

   (Electrical Engg-I, M.S. Univ. Baroda 1980)  

7. A series motor runs at 500 r.p.m. on a certain load. Calculate the resistance of a divertor required to raise the speed to 650 r.p.m. with the same load current, given that the series field resistance is 0.05 Ω and the field is unsaturated. Assume the ohmic drop in the field and armature to be negligible.  
   
   [0.1665 Ω]  

8. A 230-V d.c. series motor has armature and field resistances of 0.5 Ω and 0.3 Ω respectively. The motor draws a line current of 40 A while running at 400 r.p.m. If a divertor of resistance 0.15 W is used, find the new speed of the motor for the same armature current.  
   
   It may be assumed that flux per pole is directly proportional to the field current.  

   [1204 r.p.m.]  


9. A 250-V, d.c. shunt motor runs at 700 r.p.m. on no-load with no extra resistance in the field and armature circuit. Determine:  
   
   (i) the resistance to be placed in series with the armature for a speed of 400 r.p.m. when taking a total current of 16 A.  
   
   (ii) the resistance to be placed in series with the field to produce a speed of 1,000 r.p.m. when taking an armature current of 18 A.
Assume that the useful flux is proportional to the field. Armature resistance = 0.35 $\Omega$, field resistance = 125 $\Omega$.

10. A d.c. series motor is operating from a 220-V supply. It takes 50 A and runs at 1000 r.p.m. The resistance of the motor is 0.1 $\Omega$. If a resistance of 2 $\Omega$ is placed in series with the motor, calculate the resultant speed if the load torque is constant. [534 r.p.m.]

11. A d.c. shunt motor takes 25 A when running at 1000 r.p.m. from a 220-V supply.
   Calculate the current taken form the supply and the speed if the load torque is halved, a resistance of 5 $\Omega$ is placed in the armature circuit and a resistance of 50 $\Omega$ is placed in the field circuit.
   Armature resistance = 0.1 $\Omega$; field resistance = 100 $\Omega$
   Assume that the field flux per pole is directly proportional to the field current.[17.1 A; 915 r.p.m.]
   (Elect. Technology, Gwalior Univ. Nov. 1977)

12. A 440-V shunt motor takes an armature current of 50 A and has a flux/pole of 50 mWb. If the flux is suddenly decreased to 45 mWb, calculate (a) instantaneous increase in armature current, (b) percentage increase in the motor torque due to increase in current (c) value of steady current which motor will take eventually, (d) the final percentage increase in motor speed. Neglect brush contact drop and armature reaction and assume an armature resistance of 0.6 $\Omega$.
   [(a) 118 A (b) 112 % (c) 5.55 A (d) 10% ]

13. A 440-V shunt motor while running at 1500 r.p.m. takes an armature current of 30 A and delivers a mechanical output of 15 h.p. (11.19 kW). The load torque varies as the square of the speed. Calculate the value of resistance to be connected in series with the armature for reducing the motor speed to 1300 r.p.m. and the armature current at that speed.
   [2.97 $\Omega$, 22.5 A]

14. A 460-V series motor has a resistance of 0.4 $\Omega$ and takes a current of 25 A when there is no additional controller resistance in the armature circuit. Its speed is 1000 r.p.m. The control resistance is so adjusted as to reduce the field flux by 5%. Calculate the new current drawn by the motor and its speed. Assume that the load torque varies as the square of the speed and the same motor efficiency under the two conditions of operation.
   [22.6 A; 926 r.p.m.] (Elect. Machines, South Gujarat Univ. Oct. 1977)

15. A 460-V, series motor runs at 500 r.p.m. taking a current of 40 A. Calculate the speed and percentage change and torque if the load is reduced so that the motor is taking 30 A. Total resistance of armature and field circuit is 0.8 $\Omega$. Assume flux proportional to the field current.
   [680 r.p.m. 43.75%]

16. A 440-V, 25 h.p (18.65 kW) motor has an armature resistance of 1.2 $\Omega$ and full-load efficiency of 85%. Calculate the number and value of resistance elements of a starter for the motor if maximum permissible current is 1.5 times the full-load current.
   [1.92 $\Omega$, 1.30 $\Omega$, 0.86 $\Omega$, 0.59 $\Omega$]
   (Similar example in JNTU, Hyderabad, 2000)

17. A 230-V, d.c. shunt motor has an armature resistance of 0.3 $\Omega$. Calculate (a) the resistance to be connected in series with the armature to limit the armature current to 75 A at starting and (b) value of the generated e.m.f. when the armature current has fallen to 50 A with this value of resistance still in circuit.
   [(a) 2.767 $\Omega$ (b) 76.7 A]

18. A 200-V, d.c. shunt motor takes full-load current of 12 A. The armature circuit resistance is 0.3 $\Omega$ and the field circuit resistance is 100 $\Omega$. Calculate the value of 5 steps in the 6-stud starter for the motor. The maximum starting current is not to exceed 1.5 times the full-load current.
   [6.57 $\Omega$, 3.12 $\Omega$, 1.48 $\Omega$, 0.7 $\Omega$, 0.33 $\Omega$]

19. The resistance of a starter for a 200-V, shunt motor is such that maximum starting current is 30 A. When the current has decreased to 24 A, the starter arm is moved from the first to the second stud. Calculate the resistance between these two studs if the maximum current in the second stud is 34 A. The armature resistance of the motor is 0.4 $\Omega$.
   [1.334 $\Omega$]

20. A totally-enclosed motor has thermal time constant of 2 hr. and final temperature rise at no-load and 40° on full load.
Determine the limits between which the temperature fluctuates when the motor operates on a load cycle consisting of alternate period of 1 hr. on full-load and 1 hr. on no-load, steady state conditions having been established.  

21. A motor with a thermal time constant of 45 min. has a final temperature rise of 75°C on continuous rating (a) What is the temperature rise after one hour at this load? (b) If the temperature rise one-hour rating is 75°C, find the maximum steady temperature at this rating (c) When working at its one-hour rating, how long does it take the temperature to increase from 60°C to 75°C?  

(Electrical Technology, M.S. Univ. Baroda. 1976)

1. The speed of a d.c. motor can be controlled by varying  
   (a) its flux per pole  
   (b) resistance of armature circuit  
   (c) applied voltage  
   (d) all of the above  

2. The most efficient method of increasing the speed of a 3.75 kW d.c. shunt motor would be the ..........method.  
   (a) armature control  
   (b) flux control  
   (c) Ward-Leonard  
   (d) tapped-field control  

3. Regarding Ward-Leonard system of speed control which statement is false?  
   (a) It is usually used where wide and very sensitive speed control is required.  
   (b) It is used for motors having ratings from 750 kW to 4000 kW  
   (c) Capital outlay involved in the system is right since it uses two extra machines.  
   (d) It gives a speed range of 10 : 1 but in one direction only.  
   (e) It has low overall efficiency especially at light loads.  

4. In the rheostatic method of speed control for a d.c. shunt motor, use of armature divertor makes the method  
   (a) less wasteful  
   (b) less expensive  
   (c) unsuitable for changing loads  
   (d) suitable for rapidly changing loads  

5. The chief advantage of Ward-Leonard system of d.c. motor speed control is that it  
   (a) can be used even for small motors  
   (b) has high overall efficiency at all speeds  
   (c) gives smooth, sensitive and wide speed control  
   (d) uses a flywheel to reduce fluctuations in power demand  

6. The flux control method using paralleling of field coils when applied to a 4-pole series d.c. motor can give .......... speeds.  
   (a) 2  
   (b) 3  
   (c) 4  
   (d) 6  

7. The series-parallel system of speed control of series motors widely used in traction work gives a speed range of about  
   (a) 1 : 2  
   (b) 1 : 3  
   (c) 1 : 4  
   (d) 1 : 6  

8. In practice, regenerative braking is used when  
   (a) quick motor reversal is desired  
   (b) load has overhauling characteristics  
   (c) controlling elevators, rolling mills and printing presses etc.  
   (d) other methods can not be used.  

9. Statement 1. A direct-on-line (DOL) starter is used to start a small d.c. motor because  
   Statement 2. it limits initial current drawn by the armature circuit.  
   (a) both statement 1 and 2 are incorrect  
   (b) both statement 1 and 2 are correct  
   (c) statement 1 is correct but 2 is wrong  
   (d) statement 2 is correct but 1 is wrong  

10. Ward-Leonard system of speed control is NOT recommended for  
    (a) wide speed range  
    (b) constant-speed operation  
    (c) frequent motor reversals  
    (d) very low speeds  

11. Thyristor chopper circuits are employed for  
    (a) lowering the level of a d.c. voltage  
    (b) rectifying the a.c. voltage  
    (c) frequency conversion  
    (d) providing commutation circuitry
12. An inverter circuit is employed to convert
   (a) a.c. voltage into d.c. voltage
   (b) d.c. voltage into a.c. voltage
   (c) high frequency into low frequency
   (d) low frequency into high frequency

13. The phase-control rectifiers used for speed of
   d.c. motors convert fixed a.c. supply voltage into
   (a) variable d.c. supply voltage
   (b) variable a.c. supply voltage
   (c) full-rectified a.c. voltage
   (d) half-rectified a.c. voltage

14. If some of the switching devices in a convertor
   are controlled devices and some are diodes, the
   convertor is called
   (a) full convertor (b) semiconvertor
   (c) solid-state chopper
   (d) d.c. convertor

15. A solid-state chopper converts a fixed-voltage
   d.c. supply into a
   (a) variable-voltage a.c. supply
   (b) variable-voltage d.c. supply
   (c) higher-voltage d.c. supply
   (d) lower-voltage a.c. supply

16. The d.c. motor terminal voltage supplied by a
   solid-state chopper for speed control purposes
   varies...........with the duty ratio of the chopper
   (a) inversely  (b) indirectly
   (c) linearly  (d) parabolically

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ANSWERS

10. b   11. a
12. b   13. a   14. b   15. b  16. c